7.7 HW

1 a.) undefined at $x=0$

b.) ∞ limit

c.) ∞ limit

e.) undefined at $x=\frac{\pi}{2}$

f.) ∞ limit

i.) ∞ limit

j.) undefined at $x=0$

5. $$\int_1^\infty \frac{dx}{x^{19/20}} = \lim_{b\to\infty} \int_1^b \frac{dx}{x^{19/20}} = \lim_{b\to\infty} 20x^{\frac{1}{20}}\Big|_1^b$$

$$= \lim_{b\to\infty} 20(b)^{\frac{1}{20}} - 20 = \infty \text{ (Divergent)}$$

7. $$\int_{-\infty}^4 e^{.0001t}\,dt = \lim_{a\to-\infty} \int_a^4 e^{.0001t}\,dt = \lim_{a\to-\infty} 10{,}000\,e^{.0001t}\Big|_a^4$$

$$= \lim_{a\to-\infty} 10{,}000\left(e^{.0004} - e^{.0001a}\right) = 10{,}000\,e^{.0004} = 10{,}004.001$$

9. $$\int_0^5 \frac{dx}{x^{20/19}} = \lim_{\to 0^+} \int_a^5 \frac{dx}{x^{20/19}} = \lim_{a\to0^+} \frac{-19}{x^{1/19}}\Big|_a^5$$

$$= \lim_{a\to0^+} \frac{-19}{5^{1/19}} + \frac{19}{a^{1/19}} = \infty \text{ (Divergent)}$$

11. $$\int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{b\to4^-} \int_0^b (4-x)^{-\frac{1}{2}}dx = \lim_{b\to4^-} \left[-2\sqrt{4-x}\right]_0^b$$

$$= \lim_{b\to4^-} -2\sqrt{4-b}+2\sqrt{4-0} = 0+2(2) = 4$$

13. $$\int_2^\infty x^{-3}\,dx = \lim_{b\to\infty} \int_2^b x^{-3}dx = \lim_{b\to\infty} \left[\frac{1}{-2x^2}\right]_2^b$$

$$= \lim_{b\to\infty} \frac{1}{-2b^2} + \frac{1}{2(2)^2} = \frac{1}{8}$$

15. $\int_{-3}^{\infty} \frac{dx}{(x+4)^{3/2}} = \lim_{b\to\infty} \int_{-3}^{b} (x+4)^{-3/2}\,dx = \lim_{b\to\infty} \frac{-2}{\sqrt{x+4}}\Big|_{-3}^{b}$

$= \lim_{b\to\infty} \frac{-2}{\sqrt{b+4}} - \frac{-2}{\sqrt{-3+4}} = 0 + \frac{2}{1} = 2$

17. $\int_{0}^{1} \frac{dx}{x^{.2}} = \lim_{a\to 0^+} \int_{a}^{1} x^{-.2}\,dx = \lim_{a\to 0^+} \frac{x^{.8}}{.8}\Big|_{a}^{1}$

$= \lim_{a\to 0^+} \frac{5}{4}(1)^{.8} - \frac{5}{4}(a)^{.8} = \frac{5}{4}$

19. $\int_{4}^{\infty} e^{-3x}\,dx = \lim_{b\to\infty} \int_{4}^{b} e^{-3x}\,dx = \lim_{b\to\infty} \frac{-1}{3e^{3x}}\Big|_{4}^{b}$

$= \lim_{b\to\infty} \frac{-1}{3e^{3b}} + \frac{1}{3e^{12}} = \frac{1}{3e^{12}} = 0.00000205$

21. $\int_{-\infty}^{0} e^{3x}\,dx = \lim_{a\to-\infty} \int_{a}^{0} e^{3x}\,dx = \lim_{a\to-\infty} \frac{e^{3x}}{3}\Big|_{a}^{0}$

$= \lim_{a\to-\infty} \frac{e^{0}}{3} - \frac{e^{3a}}{3} = \frac{1}{3} - \lim_{a\to\infty} \frac{1}{3e^{3a}} = \frac{1}{3}$

23. $\int_{1}^{3} \frac{dx}{\sqrt{3-x}} = \lim_{b\to 3^-} \int_{1}^{b} (3-x)^{-\frac{1}{2}}\,dx = \lim_{b\to 3^-} \left[-2\sqrt{3-x}\right]_{1}^{b}$

$= \lim_{b\to 3^-} -2\left[\sqrt{3-b} - \sqrt{3-1}\right] = -2\left[0 - \sqrt{2}\right] = 2\sqrt{2}$

25. $\int_{0}^{\infty} \frac{dx}{1+x} = \lim_{b\to\infty} \int_{0}^{b} \frac{dx}{1+x} = \lim_{b\to\infty} \left[\ln|1+x|\right]_{0}^{b}$

$= \lim_{b\to\infty} \ln(1+\infty) - \ln 1 = \infty$ (Divergent)

29. $\int_0^{\infty} e^{-x}\cos x\,dx = \lim_{b\to\infty} \int_0^b e^{-x}\cos x\,dx$

$u=\cos x \qquad dv=e^{-x}dx$
$du=-\sin x\,dx \qquad v=-e^{-x}$

$\int e^{-x}\cos x\,dx = -e^{-x}\cos x - \int e^{-x}\sin x\,dx$ $\qquad u=\sin x \quad dv=e^{-x}dx$, $du=\cos x\,dx \quad v=-e^{-x}$

$\int e^{-x}\cos x\,dx = -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x\,dx$

$2\int e^{-x}\cos x\,dx = -e^{-x}(\cos x - \sin x) + C$

so $\lim_{b\to\infty}\int_0^b e^{-x}\cos x\,dx = \lim_{b\to\infty}\left[\frac{-e^{-x}}{2}(\cos x - \sin x)\right]_0^b$

$= 0 - (\frac{-1}{2})(1-0) = \frac{1}{2}$

33. $\int_1^{\infty} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \qquad u=\sqrt{x},\ du=\frac{1}{2\sqrt{x}}\,dx$

$= \lim_{b\to\infty} \frac{2}{1}\int_1^{\sqrt{b}} e^u\,du = \lim_{b\to\infty} 2e^u\Big|_1^{\sqrt{b}} = \infty$

35. $\int_0^{\infty}\sin x\,dx = \lim_{b\to\infty}\int_0^b \sin x\,dx = \lim_{b\to\infty}\left[-\cos x\right]_0^b$

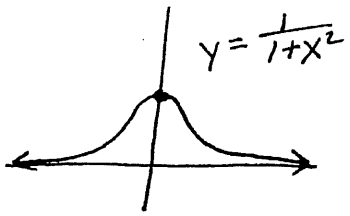
$= \lim_{b\to\infty} -\cos b + \cos(0) = 1 - \lim_{b\to\infty}\cos b$

this integral diverges, since $\lim_{b\to\infty}\cos b$ DNE

51. $\int_0^{\infty} e^{ax}\,dx = \lim_{b\to\infty}\int_0^b e^{ax}\,dx = \lim_{b\to\infty}\left[\frac{e^{ax}}{a}\right]_0^b$

$= \lim_{b\to\infty}\frac{e^{ab}}{a} - \frac{e^0}{a}$ $\qquad$ If a is positive, integral's limit is infinite. Let $a<0$ and integral converges.

If $a<0$ $\Rightarrow$ $= \lim_{b\to\infty}\frac{1}{ae^{-ab}} - \frac{1}{a} = 0 - \frac{1}{a} = \frac{-1}{a}$.

53. 

$$A = \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \lim_{a\to-\infty} \int_a^0 \frac{dx}{1+x^2} + \lim_{b\to\infty} \int_0^b \frac{dx}{1+x^2}$$

$$A = \lim_{a\to-\infty} \tan^{-1}x \Big|_a^0 + \lim_{b\to\infty} \tan^{-1}x \Big|_0^b$$

$$A = 0 - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - 0 = \pi$$

55. We know $\int_1^{\infty} \frac{1}{x^3}\,dx$ converges since $\int_1^{\infty} \frac{dx}{x^p}$ converges for $p > 1$.

Since $\frac{1}{x^3+4} \leq \frac{1}{x^3}$ (larger denominator → smaller fraction) for all $x \geq 1$ we know $\int \frac{1}{x^3+4}\,dx \leq \int \frac{1}{x^3}\,dx$. Hence $\int \frac{1}{x^3+4}\,dx$ also converges.