5. $y = 3x+1,\ [0,3] \rightarrow s = \int_0^3 \sqrt{1+(f'(x))^2}\,dx = \int_0^3 \sqrt{10}\,dx$

$f'(x) = 3$ $\qquad s = \sqrt{10}\,x \Big|_0^3 = 3\sqrt{10}$

$(f'(x))^2 = 9$

7. $y = x^{3/2},\ [1,2] \rightarrow s = \int_1^2 \sqrt{(1+\frac{9}{4}x)}\,dx = \dfrac{(1+\frac{9}{4}x)^{\frac{3}{2}}}{\frac{3}{2}(\frac{9}{4})}\Big|_1^2$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$

$(f'(x))^2 = \frac{9}{4}x$ $\qquad s = \frac{8}{27}\left(\sqrt{1+\frac{9}{4}x}\right)^3\Big|_1^2 = \frac{8}{27}\left[\left(\frac{11}{2}\right)^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right]$

9. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x,\ [1, 2e]$

$f'(x) = \frac{1}{2}x - \frac{1}{2x}$

$(f'(x))^2 = (\frac{1}{2}x - \frac{1}{2x})(\frac{1}{2}x - \frac{1}{2x}) = \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4}x^{-2}$

$1+(f'(x))^2 = \frac{1}{4}x^2 + \frac{1}{2} + \frac{1}{4}x^{-2} = (\frac{1}{2}x + \frac{1}{2}x^{-1})^2$

$s = \int_1^{2e} \sqrt{(\frac{1}{2}x + \frac{1}{2}x^{-1})^2}\,dx = \int_1^{2e} (\frac{1}{2}x + \frac{1}{2}\frac{1}{x})\,dx$

$s = \frac{1}{4}x^2 + \frac{1}{2}\ln|x|\Big|_1^{2e} = \frac{1}{4}(2e)^2 + \frac{1}{2}\ln(2e) - \frac{1}{4} - \frac{1}{2}\ln|1|$

$s = e^2 + \frac{1}{2}\ln 2 + \frac{1}{2}\ln e - \frac{1}{4} - 0 = e^2 + \frac{1}{2}\ln 2 + \frac{1}{4}$

17. $x^{2/3} + y^{2/3} = 1$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\left(\frac{dy}{dx}\right) = 0$

$\dfrac{dy}{dx} = \dfrac{-\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = \dfrac{-\sqrt[3]{y}}{\sqrt[3]{x}}$

$\left(\dfrac{dy}{dx}\right)^2 = \dfrac{y^{2/3}}{x^{2/3}}$ $\qquad$ Notice original function

$1 + \left(\dfrac{dy}{dx}\right)^2 = \dfrac{x^{2/3}}{x^{2/3}} + \dfrac{y^{2/3}}{x^{2/3}} = \dfrac{x^{2/3}+y^{2/3}}{x^{2/3}} = \dfrac{1}{x^{2/3}}$

one arc $\rightarrow s = \int_{-1}^{0} \sqrt{\dfrac{1}{x^{2/3}}}\,dx = \int_{-1}^{0} \dfrac{1}{x^{1/3}}\,dx = \frac{3}{2}x^{2/3}\Big|_{-1}^{0} = 0 + \frac{3}{2} = \frac{3}{2}$

All 4 arcs $\longrightarrow 4(\frac{3}{2}) = 6$

35. $y = x$, $[0,4]$

$\frac{dy}{dx} = 1 \rightarrow \left(\frac{dy}{dx}\right)^2 = 1$

$SA = 2\pi \int_0^4 x\sqrt{1+1}\,dx$

$SA = 2\sqrt{2}\pi \left[\frac{x^2}{2}\right]_0^4 = \sqrt{2}\pi x^2 \Big|_0^4$

$SA = 16\sqrt{2}\pi$

37. $y = x^3, [0,2] \rightarrow SA = 2\pi \int_0^2 x^3\sqrt{1+9x^4}\,dx$

$u = 1+9x^4$
$du = 36x^3\,dx$
$u(0) = 1$
$u(2) = 145$

$\frac{dy}{dx} = 3x^2$

$\left(\frac{dy}{dx}\right)^2 = 9x^4$

$SA = \frac{\pi}{18}\int_1^{145} u^{\frac{1}{2}}\,du$

$SA = \frac{\pi}{18}\left(\frac{2}{3}\right)u^{3/2}\Big|_1^{145} = \frac{\pi}{27}\left(\sqrt{u}\right)^3\Big|_1^{145}$

$SA = \frac{\sqrt{(145)^3}\,\pi - \pi}{27} = \frac{\left(\sqrt{(145)^3} - 1\right)\pi}{27}$

41. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x,\ [1,e] \rightarrow SA = 2\pi \int_1^e \left(\frac{1}{4}x^2 - \frac{1}{2}\ln x\right)\sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\,dx$

$\frac{dy}{dx} = \frac{1}{2}x - \frac{1}{2x}$

$SA = 2\pi \int_1^e \left(\frac{1}{4}x^2 - \frac{1}{2}\ln x\right)\left(\frac{1}{2}x + \frac{1}{2x}\right)dx$

$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}$

$SA = 2\pi \int_1^e \left(\frac{1}{8}x^3 + \frac{1}{8}x - \frac{1}{4}x\ln x - \frac{1}{4x}\ln x\right)dx$

$1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}x^2 + \frac{1}{2} + \frac{1}{4x^2}$

$1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{2}x + \frac{1}{2x}\right)^2$

easy IBP sub

$SA = 2\pi \int_1^e \left(\frac{1}{8}x^3 + \frac{1}{8}x\right)dx - \frac{\pi}{2}\int_1^e x\ln x\,dx - \frac{\pi}{2}\int_1^e \frac{\ln x}{x}\,dx$

$u = \ln x \quad dv = x\,dx$
$du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$t = \ln x$
$dt = \frac{1}{x}dx$

$SA = 2\pi\left[\frac{x^4}{32} + \frac{x^2}{16}\right]_1^e - \frac{\pi}{2}\left[\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right]_1^e - \frac{\pi}{2}\left[\int t\,dt\right]_{x=1}^{x=e}$

$SA = 2\pi\left[\frac{e^4}{32} + \frac{e^2}{16} - \frac{1}{32} - \frac{1}{16}\right] - \frac{\pi}{2}\left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_1^e - \frac{\pi}{2}\left[\frac{(\ln x)^2}{2}\right]_1^e$

$SA = 2\pi\left[\frac{e^4+2e^2-3}{32}\right] - \frac{\pi}{2}\left[\frac{1}{4}e^2 + \frac{1}{4}\right] - \frac{\pi}{2}\left[\frac{1}{2}\right] = \frac{(e^4+2e^2-3)\pi}{16} - \frac{2\pi e^2}{16} - \frac{3\pi}{8}$

$SA = \frac{(e^4-9)\pi}{16}$