The Definite Integral

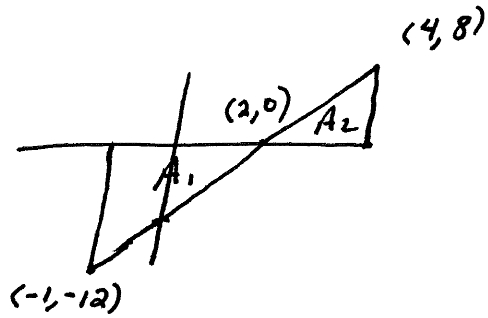
1. $\int_{-1}^{4}(4x-8)\,dx = R_n = \lim_{n\to\infty}\sum f(a+i\Delta x)\Delta x = \lim_{n\to\infty}\sum f\left(-1+\frac{5i}{n}\right)\left(\frac{5}{n}\right)$

$$= \lim_{n\to\infty}\frac{5}{n}\sum\left[4\left(-1+\frac{5i}{n}\right)-8 = \lim_{n\to\infty}\frac{5}{n}\sum\left(-12+\frac{20i}{n}\right)\right.$$

$$= \lim_{n\to\infty}\frac{5}{n}(-12n) + \frac{5}{n}\left(\frac{20}{n}\right)\sum i$$

$$= -60 + \lim_{n\to\infty}\frac{100}{n^2}\left(\frac{n(n+1)}{2}\right) = -60+50 = -10$$

b.) Graphically

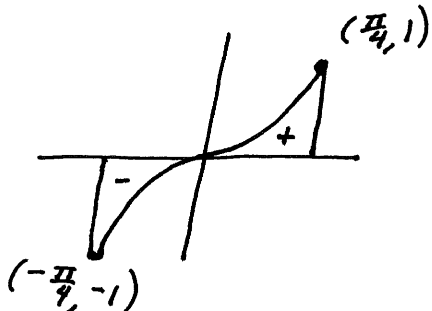


$$\int_{-1}^{4}(4x-8)\,dx = A_1 + A_2$$

$$= \tfrac{1}{2}(3)(-12) + \tfrac{1}{2}(2)(8)$$

$$= -18+8 = -10$$

2. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x\,dx$



Since $y=\tan x$ is an odd function ($f(-x)=-f(x)$),

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\tan x\,dx = \int_{-\frac{\pi}{4}}^{0}\tan x\,dx + \int_{0}^{\frac{\pi}{4}}\tan x\,dx = 0.$$

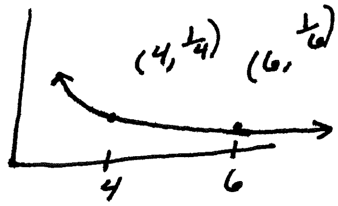
3. $\int_{2}^{9} f(x)\,dx - \int_{4}^{9} f(x)\,dx = \int_{2}^{4} f(x)\,dx$

4. $|2x-4| = \begin{cases} 2x-4, & x \geq 2 \\ -(2x-4), & x < 2 \end{cases}$ so $\int_1^3 |2x-4|\,dx = -\int_1^2 (2x-4)\,dx + \int_2^3 (2x-4)\,dx$

$$\int_1^3 |2x-4|\,dx = \left[x^2-4x\right]_2^1 + \left[x^2-4x\right]_2^3$$

$$= (-3-(-4)) + [-3-(-4)] = 1+1 = 2$$

5. Prove that $\frac{1}{3} < \int_4^6 \frac{1}{x}\,dx \leq \frac{1}{2}$



Since $y = \frac{1}{x}$ is decreasing on $[4, 6]$ we know

$$R_1 < \int_4^6 \frac{1}{x}\,dx < L_1$$

$$\frac{1}{6}(2) < \int_4^6 \frac{1}{x}\,dx < \frac{1}{4}(2)$$

$$\frac{1}{3} < \int_4^6 \frac{1}{x}\,dx < \frac{1}{2}$$

QED