Definite Integrals

1. $\int_4^1 t^{5/2}\,dt = \left.\frac{t^{\frac{7}{2}}}{\frac{7}{2}}\right|_4^1 = \left.\frac{2}{7}t^{\frac{7}{2}}\right|_4^1$

$= \frac{2}{7}(1-128) = \frac{-254}{7}$

Note: $\int_4^1 t^{5/2}\,dt = -\int_1^4 t^{5/2}\,dt = -\left[\frac{2}{7}t^{\frac{7}{2}}\right]_1^4$

$= \frac{-2}{7}\left[128-1\right] = \frac{-254}{7}$

2. $\int_{\frac{8}{27}}^{1} \frac{10t^{4/3}-8t^{1/3}}{t^2}\,dt = \int_{\frac{8}{27}}^{1}(10t^{-2/3}-8t^{-5/3})\,dt$

$= \left[\frac{10t^{1/3}}{\frac{1}{3}} - \frac{8t^{-2/3}}{-2/3}\right]_{\frac{8}{27}}^{1} = \left[30t^{1/3}+\frac{12}{t^{2/3}}\right]_{\frac{8}{27}}^{1}$

$= (30+12) - \left(30\left(\frac{2}{3}\right) + \frac{12}{\frac{4}{9}}\right) = 42-(20+27) = -5$

3. $\int_{\frac{\pi}{28}}^{\frac{\pi}{14}} \csc^2(7y)\,dy$ ⟵ since $\frac{d}{dx}(-\cot x) = \csc^2 x$

$= \left[\frac{-\cot(7y)}{7}\right]_{\frac{\pi}{28}}^{\frac{\pi}{14}} = \frac{-\cot\left(\frac{\pi}{2}\right)}{7} + \frac{\cot\left(\frac{\pi}{4}\right)}{7}$

$= 0 + \frac{1}{7} = \frac{1}{7}$

4. $\displaystyle\int_0^5 |3-x|\,dx$ $\qquad |3-x| = \begin{cases} 3-x, & x \le 3 \\ -(3-x), & x > 3 \end{cases}$

$$= \int_0^3 (3-x)\,dx - \int_3^5 (3-x)\,dx$$

$$= \left[3x - \frac{x^2}{2}\right]_0^3 - \left[3x - \frac{x^2}{2}\right]_3^5$$

$$= \left(9 - \frac{9}{2}\right) - 0 - \left[\left(15 - \frac{25}{2}\right) - \left(\frac{9}{1} - \frac{9}{2}\right)\right]$$

$$= \frac{9}{2} - \left(\frac{5}{2} - \frac{9}{2}\right) = \frac{13}{2}$$

5. $\displaystyle\int_{-x}^{x} (t^3+t)\,dt = \left[\frac{t^4}{4} + \frac{t^2}{2}\right]_{-x}^{x}$

$$= \left(\frac{1}{4}x^4 + \frac{1}{2}x^2\right) - \left(\frac{1}{4}x^4 + \frac{1}{2}x^2\right) = 0$$