Substitution

1. $\int \frac{2x^2+x}{(4x^3+3x^2)^2}\,dx$ $\qquad u = 4x^3+3x^2$

$\qquad du = (12x^2+6x)\,dx = 6(2x^2+x)\,dx$

$= \frac{1}{6}\int \frac{6(2x^2+x)\,dx}{(4x^3+3x^2)^2} = \frac{1}{6}\int \frac{1}{u^2}\,du = \frac{1}{6}\int u^{-2}\,du = \frac{1}{6}\left[\frac{u^{-1}}{-1}+C\right]$

$= \frac{-1}{6u} + C = \frac{-1}{6(4x^3+3x^2)} + C$

2. $\int x(3x+8)^{11}\,dx$ $\qquad u = 3x+8 \rightarrow u-8 = 3x \rightarrow x = \frac{1}{3}(u-8)$

$\qquad du = 3\,dx$

$= \frac{1}{3}\int \frac{1}{3}(u-8)(u)^{11}(3\,dx) = \frac{1}{9}\int (u-8)u^{11}\,du = \frac{1}{9}\int (u^{12}-8u^{11})\,du$

$= \frac{1}{9}\left[\frac{1}{13}u^{13} - \frac{2}{3}u^{12} + C\right] = \frac{(3x+8)^{13}}{117} - \frac{2(3x+8)^{12}}{27} + C$

3. $\int \theta \sin(\theta^2)\,d\theta$ $\qquad u = \theta^2$

$\qquad du = 2\theta\,d\theta$

$= \frac{1}{2}\int 2\theta \sin(\theta^2)\,d\theta = \frac{1}{2}\int \sin u\,du = \frac{1}{2}\left[-\cos u + C\right]$

$= \frac{-1}{2}\cos(\theta^2) + C$

4. $\int \cos t \cos(\sin t)\,dt$ $\qquad u = \sin t$

$\qquad du = \cos t\,dt$

$= \int \cos u\,du = \sin u + C = \sin(\sin t) + C$

5. $\int_1^2 \frac{4X+12}{(X^2+6X+1)^2}\,dX$

$u = X^2+6X+1 \qquad u(1) = 1^2+6+1 = 8$

$du = (2X+6)\,dX \qquad u(2) = 2^2+12+1 = 17$

$= \int_1^2 \frac{2(2X+6)}{(X^2+6X+1)^2}\,dX = 2\int_8^{17} \frac{1}{u^2}\,du = 2\int_8^{17} u^{-2}\,du = 2\left[\frac{-1}{u}\right]_8^{17}$

$= 2\left[\frac{-1}{17} + \frac{1}{8}\right] = 2\left[\frac{-8}{136} + \frac{17}{136}\right] = \frac{9}{68}$