Chapter 4
Probability Distributions

4-1 Overview

4-2 Random Variables

4-3 Binomial Probability Distributions

4-4 Mean, Variance, Standard Deviation for the Binomial Distribution

Overview

This chapter will deal with the construction of

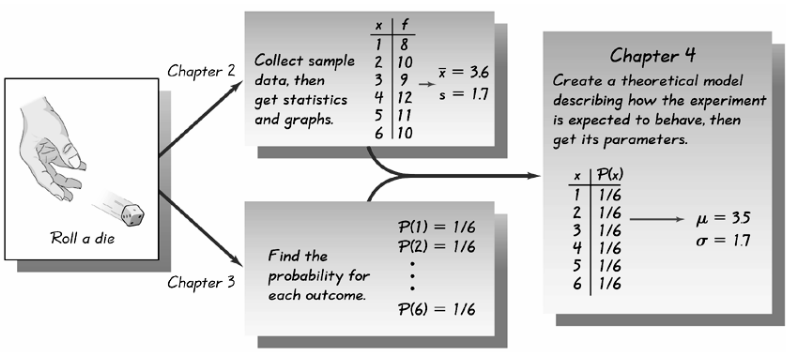
probability distributions

by combining the methods of Chapter 2 with the those of Chapter 3.

Probability Distributions will describe what will *probably* happen instead of what actually *did* happen.

Combining Descriptive Statistics Methods and Probabilities to Form a Theoretical Model of Behavior

Figure 4-1



4-2
Random Variables

Definitions

❖ **Random Variable**

a variable (typically represented by x) that has a single numerical value, determined by chance, for each outcome of a procedure

❖ **Probability Distribution**

a graph, table, or formula that gives the probability for each value of the random variable

Table 4-1

Probability Distribution
Number of Girls Among Fourteen Newborn Babies

x	$P(x)$
0	0.000
1	0.001
2	0.006
3	0.022
4	0.061
5	0.122
6	0.183
7	0.209
8	0.183
9	0.122
10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

Definitions

❖ **<u>Discrete</u> random variable**

has either a finite number of values or countable number of values, where 'countable' refers to the fact that there might be infinitely many values, but they result from a counting process.

❖ **<u>Continuous</u> random variable**

has infinitely many values, and those values can be associated with measurements on a continuous scale with no gaps or interruptions.

Probability Histogram

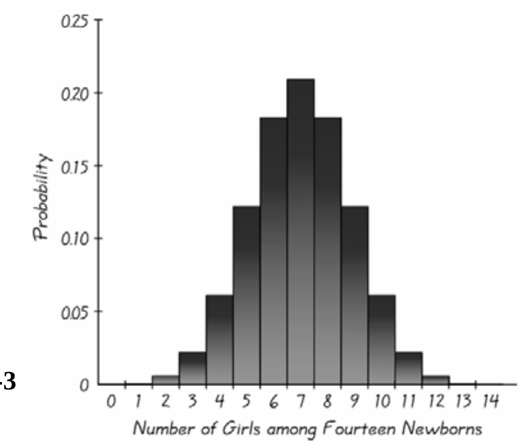


Figure 4-3

Requirements for Probability Distribution

$$\sum P(x) = 1$$

where x assumes all possible values

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$$\sum P(x) = 1$$

where x assumes all possible values

$$0 \leq P(x) \leq 1$$

for every value of x

Mean, Variance and Standard Deviation of a Probability Distribution

Formula 4-1

$$\mu = \Sigma [x \cdot P(x)]$$

Formula 4-2

$$\sigma^2 = \Sigma [(x - \mu)^2 \cdot P(x)]$$

Formula 4-3

$$\sigma^2 = [\Sigma x^2 \cdot P(x)] - \mu^2 \quad \text{(shortcut)}$$

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$$\sigma^2 = [\Sigma x^2 \cdot P(x)] - \mu^2 \quad \text{(shortcut)}$$

Formula 4-4

$$\sigma = \sqrt{[\Sigma x^2 \cdot P(x)] - \mu^2}$$

Usual Sample Values

$$\text{minimum} = \mu - 2(\sigma)$$

$$\text{maximum} = \mu + 2(\sigma)$$

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11	0.022
12	0.006
13	0.001
14	0.000

Usual Sample Values

$$\text{minimum} = 7.0 - 2(1.876) = 3.248$$

$$\text{maximum} = 7.0 + 2(1.876) = 10.752$$

Using the Rare Event Rule

If, under a given assumption (such as the assumption that boys and girls are equally likely), the probability of a particular observed event (such as 13 girls in 14 births) is extremely small, we conclude that the assumption was probably not correct (boys and girls NOT equally likely; the gender selection technique did have an effect).

Using Probabilities to Determine When Results Are Unusual

X is unusually high if with *x* successes among *n* trials, P(x or more) is very small (such as 0.05 or less)

X is unusually low if with *x* successes among *n* trials, P(x or fewer) is very small (such as 0.05 or less)

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5	0.122
6	0.183
7	0.209
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9	0.122
10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

Definition

Expected Value

The average value of outcomes

$$E = \Sigma [x \cdot P(x)]$$

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Event	x	P(x)
Win	$499	0.001
Lose	- $1	0.999

$$E = \Sigma [x \cdot P(x)]$$

Event	x	P(x)	x • P(x)
Win	$499	0.001	0.499
Lose	- $1	0.999	- 0.999

$$E = \Sigma [x \cdot P(x)]$$

Event	x	P(x)	x • P(x)
Win	$499	0.001	0.499
Lose	- $1	0.999	- 0.999

E = -$.50