

4-4

Mean, Variance, Standard Deviation for Binomial Distributions

For Any Discrete Probability Distribution:

Formula 4-1 $\mu = \sum[x \cdot P(x)]$

Formula 4-3 $\sigma^2 = [\sum x^2 \cdot P(x)] - \mu^2$

Formula 4-4 $\sigma = \sqrt{[\sum x^2 \cdot P(x)] - \mu^2}$
or use calculator

Table 4-1

Probability Distribution Number of Girls Among Fourteen Newborn Babies

x	$P(x)$
0	0.000
1	0.001
2	0.006
3	0.022
4	0.061
5	0.122
6	0.183
7	0.209
8	0.183
9	0.122
10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

For Binomial Distributions:

Formula 4-6 $\mu = n \cdot p$

Formula 4-7 $\sigma^2 = n \cdot p \cdot q$

Formula 4-8 $\sigma = \sqrt{n \cdot p \cdot q}$

Example: Find the mean and standard deviation for the number of girls in groups of 14 births.

We previously discovered that this scenario could be considered a binomial experiment where:

$$n = 14$$

$$p = 0.5$$

$$q = 0.5$$

Using the binomial distribution formulas:

Example: Find the mean and standard deviation for the number of girls in groups of 14 births.

We previously discovered that this scenario could be considered a binomial experiment where:

$$n = 14$$

$$p = 0.5$$

$$q = 0.5$$

Using the binomial distribution formulas:

$$\mu = (14)(0.5) = 7 \text{ girls}$$

$$\sigma = \sqrt{(14)(0.5)(0.5)} = 1.9 \text{ girls (rounded)}$$

Reminder

$$\text{Minimum usual values} = \mu - 2 \sigma$$

$$\text{Maximum usual values} = \mu + 2 \sigma$$

Example: Determine whether 12 girls among 14 births could easily occur by chance.

For this binomial distribution,

$$\mu = 7 \text{ girls}$$

$$\sigma = 1.9 \text{ girls}$$

$$\mu - 2 \sigma = 7 - 2(1.9) = 3.2$$

$$\mu + 2 \sigma = 7 + 2(1.9) = 10.8$$

The usual number girls among 14 births would be from 4 to 10. So 12 girls in 14 births is an unusual result.

Using Probabilities to Determine When Results Are Unusual

X is unusually high if with x successes among n trials, $P(x \text{ or more})$ is very small (such as 0.05 or less or is 2 SD above mean)

X is unusually low if with x successes among n trials, $P(x \text{ or fewer})$ is very small (such as 0.05 or less or is two SD below mean)