

Chapter 5

Normal Probability Distributions

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Normal Probability Distributions

5-1 Overview

5-2 The Standard Normal Distribution

5-3 Applications of Normal Distributions

5-2 and 5-3: Normal Distributions: Finding Values

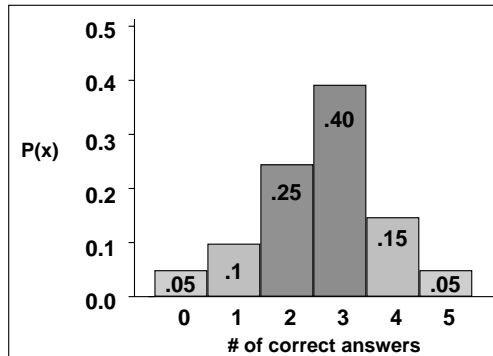
5-5 The Central Limit Theorem

1

Review

x (# of correct)	$P(x)$
0	.05
1	.10
2	.25
3	.40
4	.15
5	.05

Probability Distribution



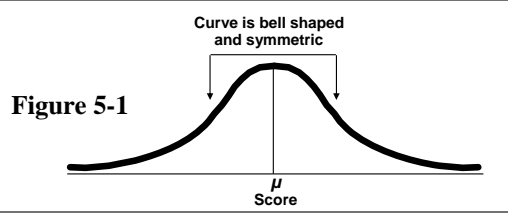
Probability Histogram

Overview

- ❖ Continuous random variable
- ❖ Normal distribution

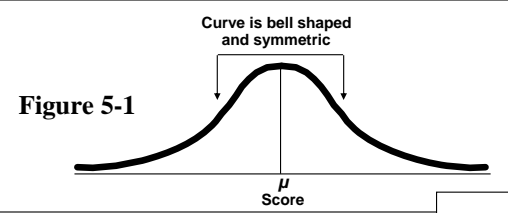
Overview

- ❖ Continuous random variable
- ❖ Normal distribution



Overview

- ❖ Continuous random variable
- ❖ Normal distribution



Formula 5-1

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma \sqrt{2\pi}}$$

2

5-2

The Standard Normal Distribution

Definitions

- ❖ **Density Curve**
(or probability density function)
the graph of a continuous probability distribution
1. The total area under the curve must equal 1.
 2. Every point on the curve must have a vertical height that is 0 or greater.

Because the total area under the density curve is equal to 1, there is a correspondence between area and probability.

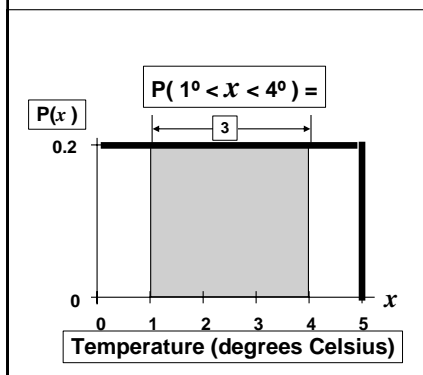
Definition

❖ Uniform Distribution

a probability distribution in which the continuous random variable values are spread evenly over the range of possibilities; the graph results in a rectangular shape.

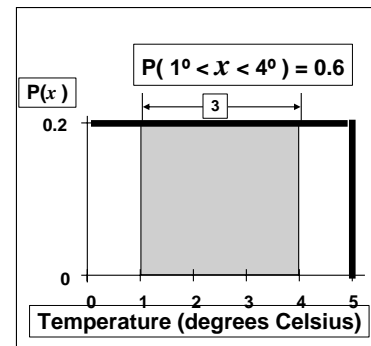
3

Uniform Distributions

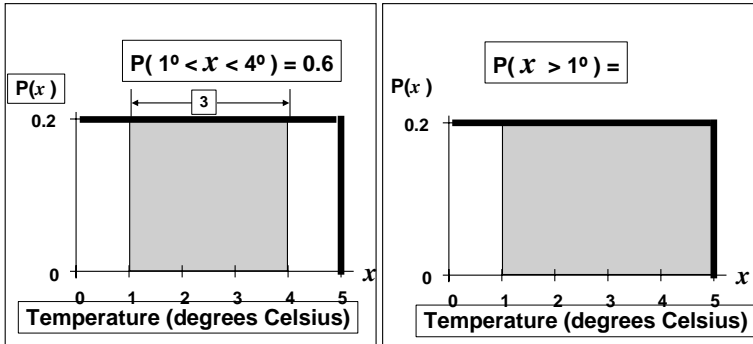


Uniform Distributions

$$\text{Shaded Area} = L \cdot W = 3 \cdot 0.2 = 0.6$$

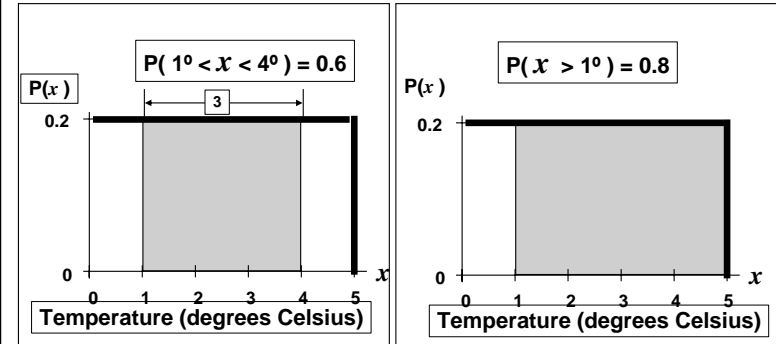


Uniform Distributions



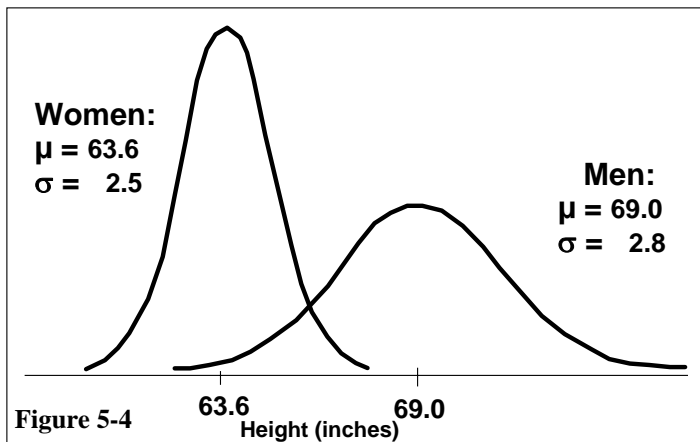
Uniform Distributions

Shaded Area = $L \cdot W = 4 \cdot 0.2 = 0.8$

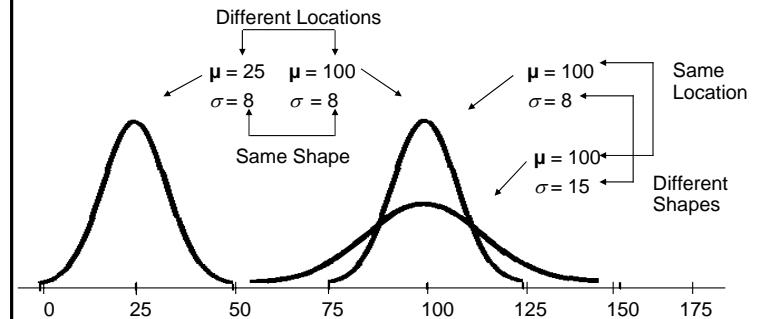


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Heights of Adult Men and Women



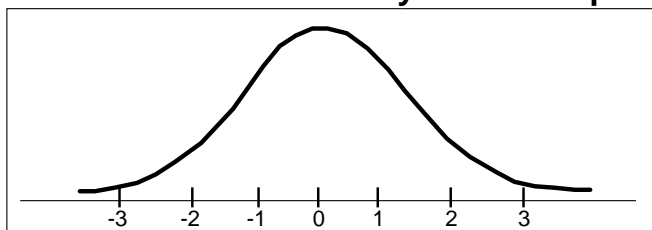
Normal Distributions



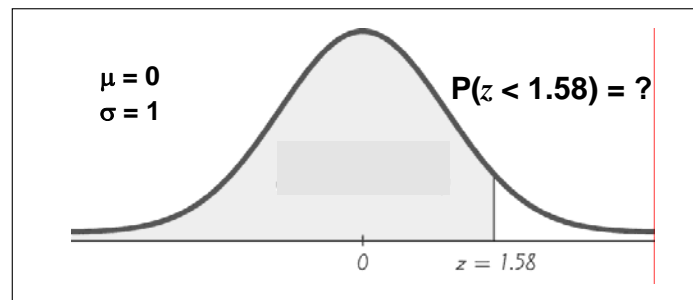
Definition

Standard Normal Distribution

a normal probability distribution that has a mean of 0 and a standard deviation of 1, and the total area under its density curve is equal to 1.



Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



5

Methods for Finding Normal Distribution Areas

- ❖ Table A-2 (sometimes called z-distribution or z-table)
- ❖ Computer programs
- ❖ TI-83 Plus Calculator

Table A-2

- ❖ Front left cover of text book
- ❖ Formulas and Tables card
- ❖ Appendix

NEGATIVE Z Scores Table A-2

TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.50 and lower	.0001									
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

Table A-2

- ❖ Designed only for *standard* normal distribution
- ❖ Is on two pages: *negative z-scores* and *positive z-scores*
- ❖ Body of table is a cumulative area from the left up to a vertical boundary
- ❖ Avoid confusion between z-scores and areas
- ❖ Z-score hundredths is across the top row

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Table A-2 Standard Normal Distribution

Negative z-scores: cumulative from left

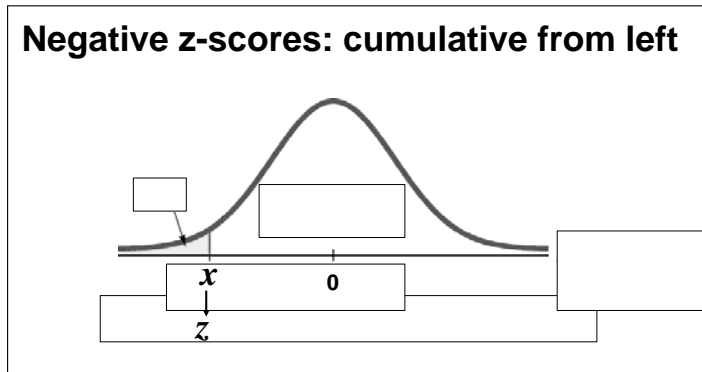
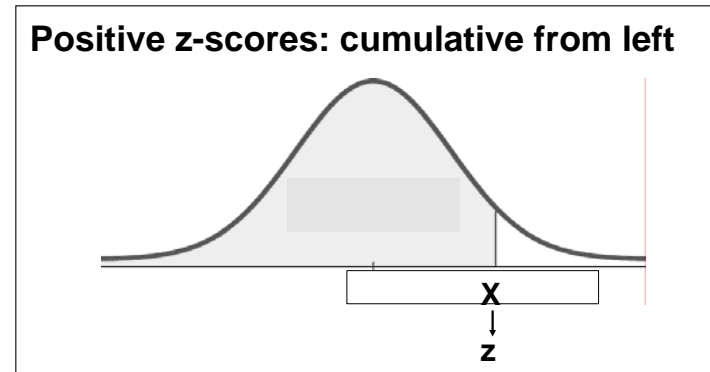
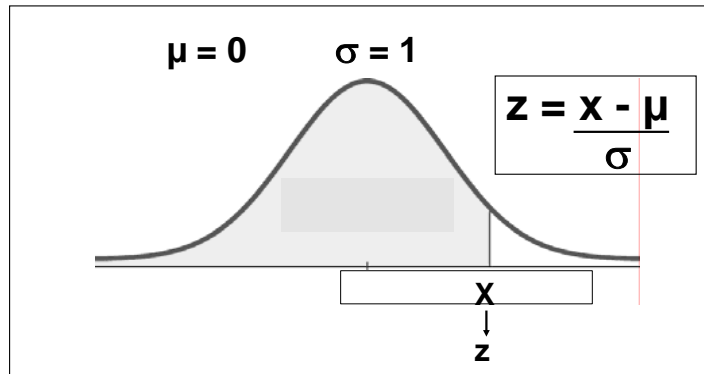


Table A-2 Standard Normal Distribution

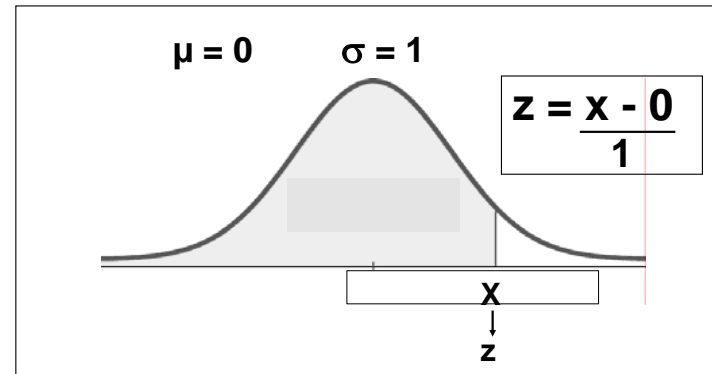
Positive z-scores: cumulative from left



**Table A-2
Standard Normal Distribution**

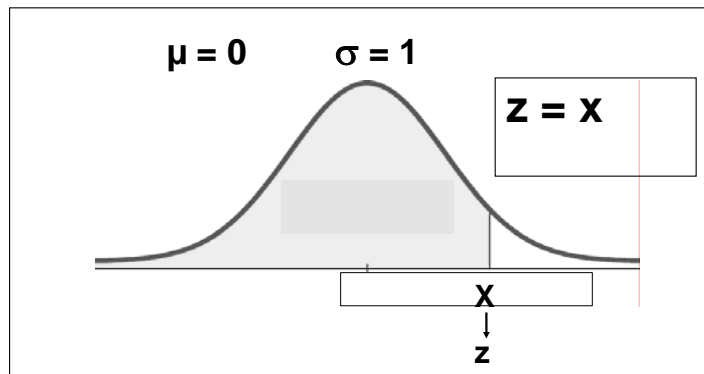


**Table A-2
Standard Normal Distribution**

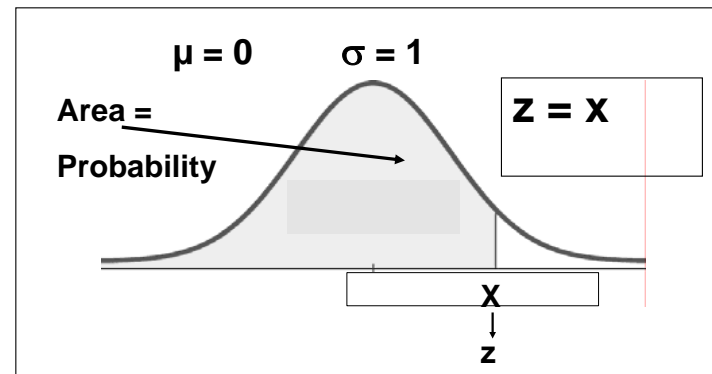


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**Table A-2
Standard Normal Distribution**



**Table A-2
Standard Normal Distribution**



To find:

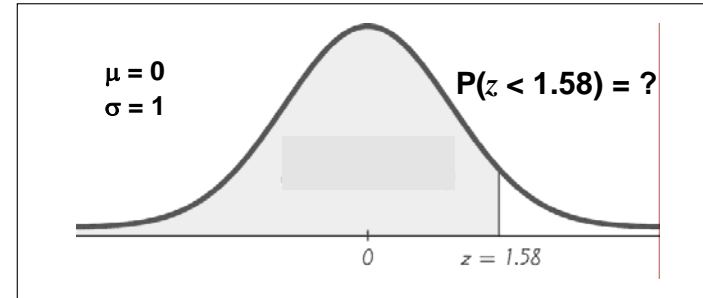
z Score

the distance along horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table A-2

Area

the region under the curve; refer to the values in the body of Table A-2

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.

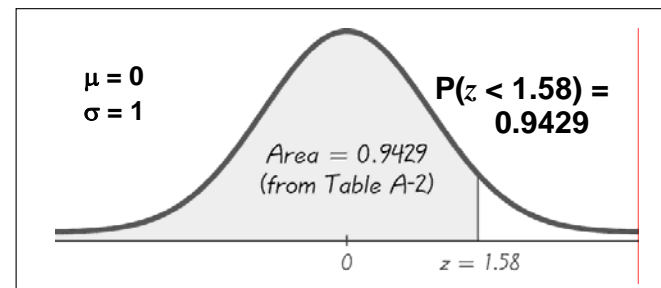


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POSITIVE z Scores

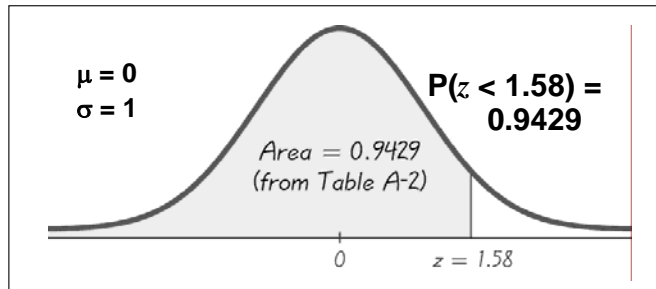
TABLE A-2 (continued) Cumulative Area from the LEFT										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



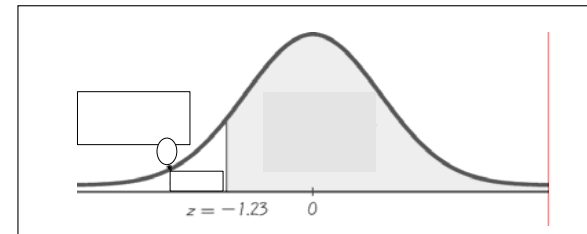
The probability that the chosen thermometer will measure freezing water less than 1.58 degrees is 0.9429.

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



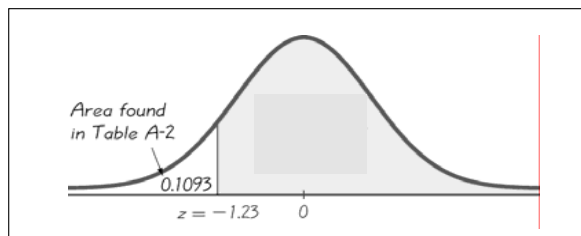
94.29% of the thermometers will read freezing water less than 1.58 degrees.

Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = ?$

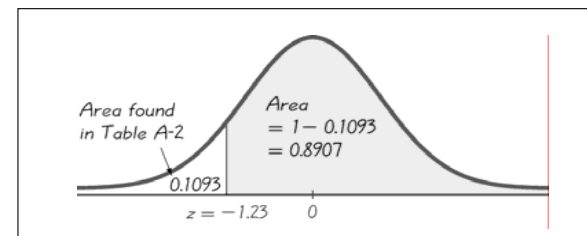


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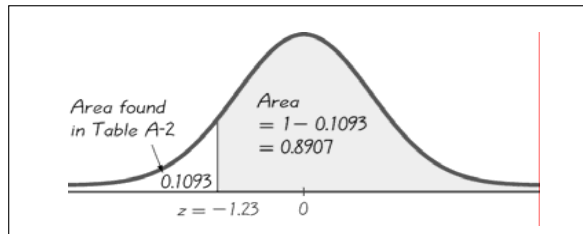
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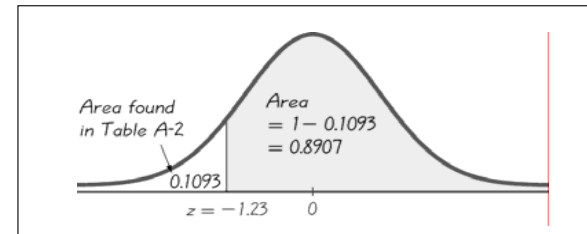
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Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = 0.8907$



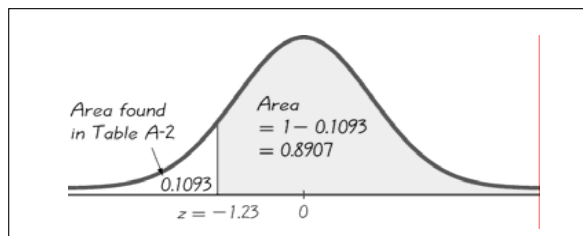
Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = 0.8907$



The probability that the chosen thermometer with a reading above -1.23 degrees is 0.8907.

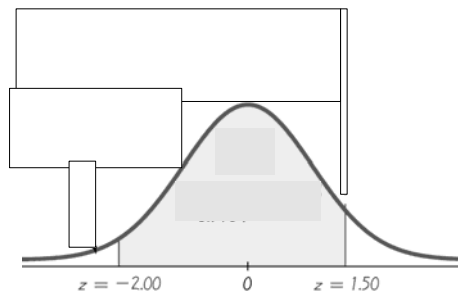
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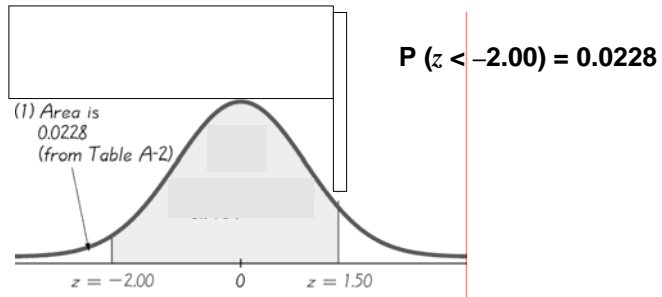


The percentage of thermometers with a reading above -1.23 degrees is 89.07%.

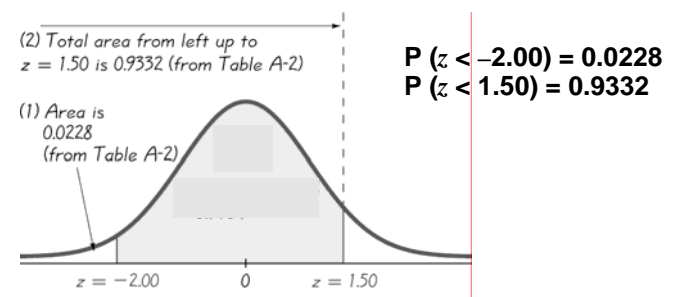
Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.



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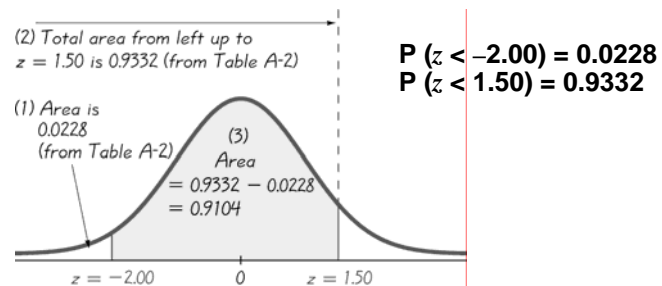


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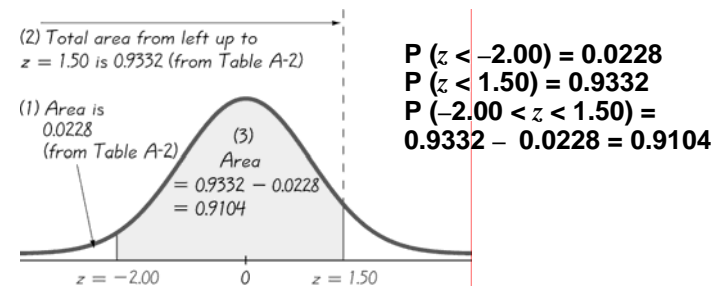


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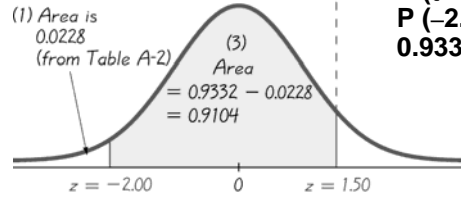


Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.

(2) Total area from left up to $z = 1.50$ is 0.9332 (from Table A-2)

(1) Area is 0.0228 (from Table A-2)

(3) Area = $0.9332 - 0.0228 = 0.9104$



$$\begin{aligned} P(z < -2.00) &= 0.0228 \\ P(z < 1.50) &= 0.9332 \\ P(-2.00 < z < 1.50) &= 0.9332 - 0.0228 = 0.9104 \end{aligned}$$

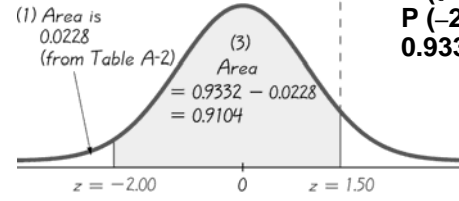
The probability that the chosen thermometer has a reading between -2.00 and 1.50 degrees is 0.9104 .

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.

(2) Total area from left up to $z = 1.50$ is 0.9332 (from Table A-2)

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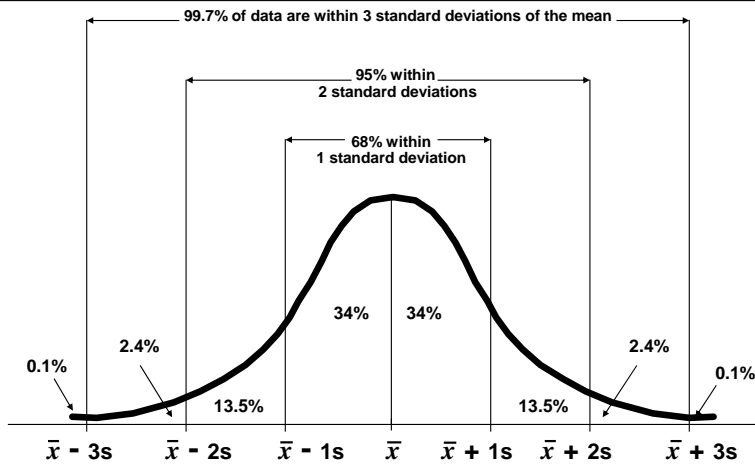
$$\begin{aligned} P(z < -2.00) &= 0.0228 \\ P(z < 1.50) &= 0.9332 \\ P(-2.00 < z < 1.50) &= 0.9332 - 0.0228 = 0.9104 \end{aligned}$$

The percentage of thermometers that read between -2.00 and 1.50 degrees is 91.04% .

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The Empirical Rule

Standard Normal Distribution: $\mu = 0$ and $\sigma = 1$



Notation

$$P(a < z < b)$$

denotes the probability that the z score is between a and b

$$P(z > a)$$

denotes the probability that the z score is greater than a

$$P(z < a)$$

denotes the probability that the z score is less than a