

Measures of Variation

Section 2-5

Waiting Times of Bank Customers at Different Banks in minutes

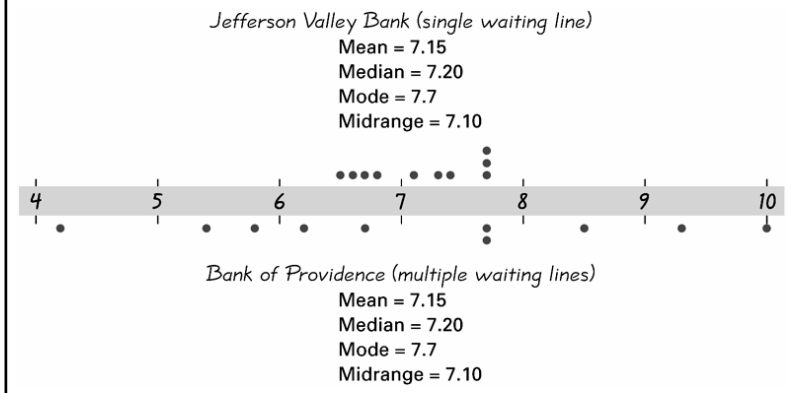
Jefferson Valley Bank	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
Bank of Providence	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0

Waiting Times of Bank Customers at Different Banks in minutes

Jefferson Valley Bank	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
Bank of Providence	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0

	Jefferson Valley Bank	Bank of Providence
Mean	7.15	7.15
Median	7.20	7.20
Mode	7.7	7.7
Midrange	7.10	7.10

Dotplots of Waiting Times



Measures of Variation

Range

$$\text{highest value} - \text{lowest value}$$

Measures of Variation

Standard Deviation

- ❖ A measure of variation of the scores about the mean
- ❖ Average deviation from the mean
- ❖ Average 'distance' scores are from the mean

Standard Deviation Formula for a Sample

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

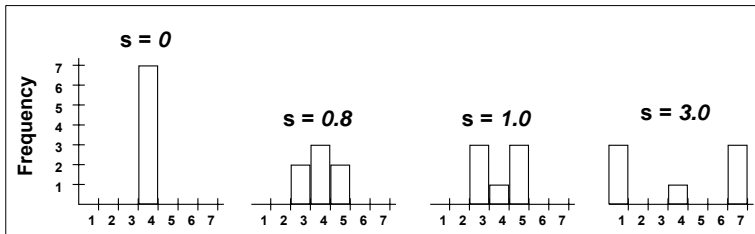
Formula 2-4

Compute the standard deviation for the numbers 1,2,3,4,5 by hand and by calculator.

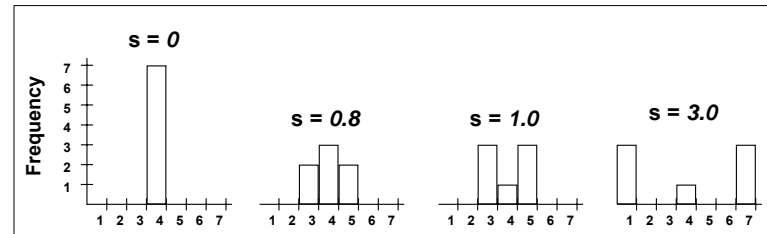
Important Properties of Standard Deviation

- ❖ A measure of variation of all values from the mean
- ❖ Usually positive; is zero (0) when all data are the same
- ❖ Value can increase dramatically with outliers
- ❖ Units are the same as the units of the original data

**Same Means ($\bar{x} = 4$)
Different Standard Deviations**



**Same Means ($\bar{x} = 4$)
Different Standard Deviations**



When data is more varied, the standard deviation gets larger.

**Using Your Calculator to find
the standard deviation of a
data set**

6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7

**Using Your Calculator to find
the standard deviation of a
data set**

6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7

S = 0.48 minutes

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

calculators can compute the
population standard deviation
of data

Measures of Variation

Variance

Measures of Variation

Variance

standard deviation squared

Measures of Variation

Variance

standard deviation squared

Notation $\left\{ \begin{array}{l} S^2 \\ \sigma^2 \end{array} \right.$ use square key
on calculator

Variance

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$
 Sample Variance

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N}$$
 Population Variance

Using Your Calculator to find the standard deviation of a data set

6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7

S = 0.48 minutes

Using Your Calculator to find the standard deviation of a data set

6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7

S = 0.48 minutes

S² = 0.23 minutes²

Notation

Sample

Textbook → **S**
Some graphics calculators → **Sx**
Some non-graphics calculators → **Xσ_{n-1}**

Notation

Sample

Population

Textbook → S	σ ← Book
Some graphics calculators → Sx	σx ← Some graphics calculators
Some non-graphics calculators → $x\sigma_{n-1}$	$x\sigma_n$ ← Some non-graphics calculators

Notation

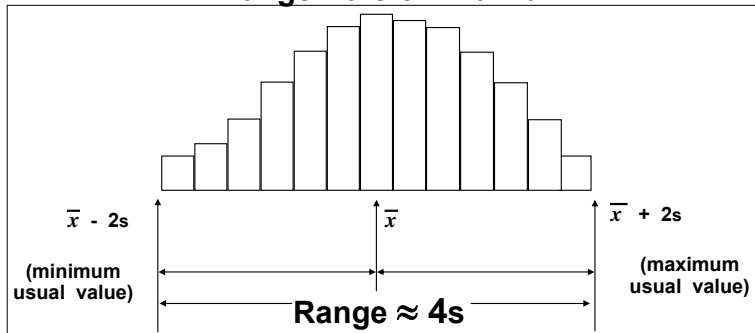
Sample

Population

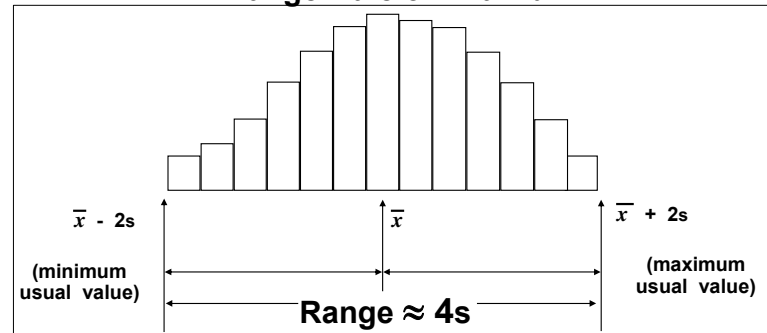
Textbook → S	σ ← Book
Some graphics calculators → Sx	σx ← Some graphics calculators
Some non-graphics calculators → $x\sigma_{n-1}$	$x\sigma_n$ ← Some non-graphics calculators

Articles in professional journals and reports often use SD for standard deviation and VAR for variance.

Estimation of Standard Deviation Range Rule of Thumb

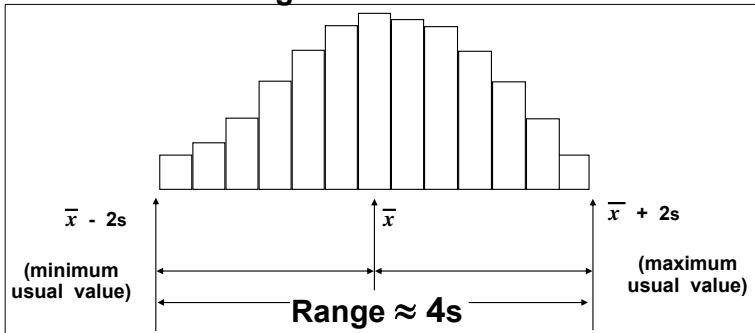


Estimation of Standard Deviation Range Rule of Thumb



$$s \approx \frac{\text{Range}}{4}$$

Estimation of Standard Deviation Range Rule of Thumb



$$s \approx \frac{\text{Range}}{4} = \frac{\text{highest value} - \text{lowest value}}{4}$$

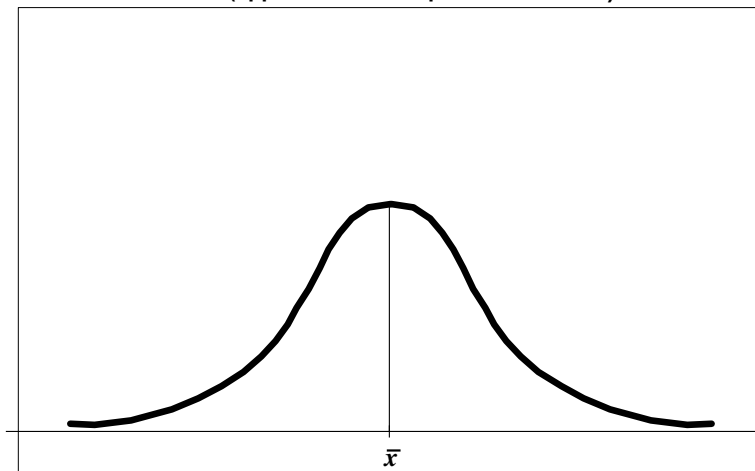
Estimating the standard deviation using the Range Rule of Thumb

6.5	6.6	6.7	6.8	7.1
7.3	7.4	7.7	7.7	7.7

$$\begin{aligned}
 S &\approx \text{Range} / 4 = (7.7 - 6.5) / 4 \\
 &= 1.2 / 4 \\
 &= 0.3 \text{ min (estimate)} \\
 S &= 0.48 \text{ minutes (actual)}
 \end{aligned}$$

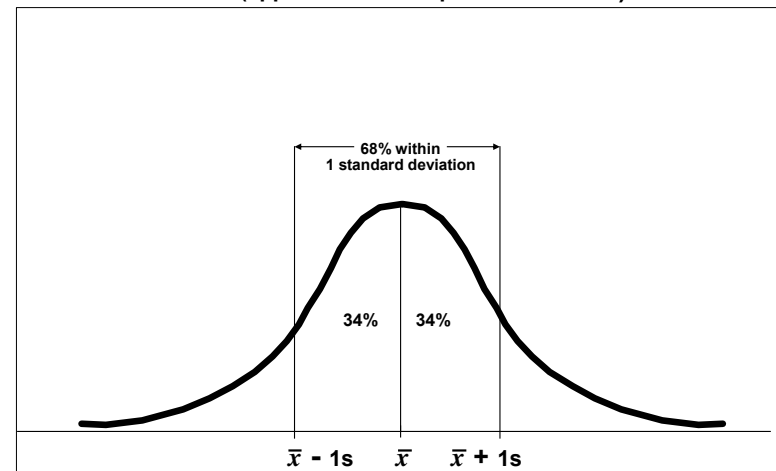
The Empirical Rule

FIGURE 2-13 (applies to bell-shaped distributions)



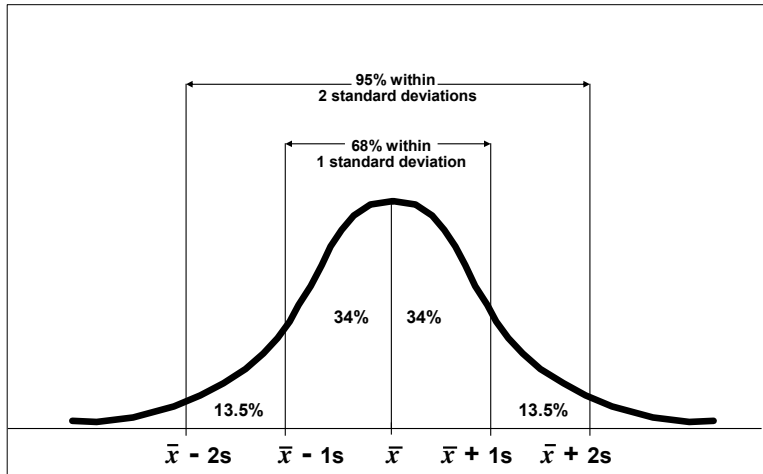
The Empirical Rule

FIGURE 2-13 (applies to bell-shaped distributions)



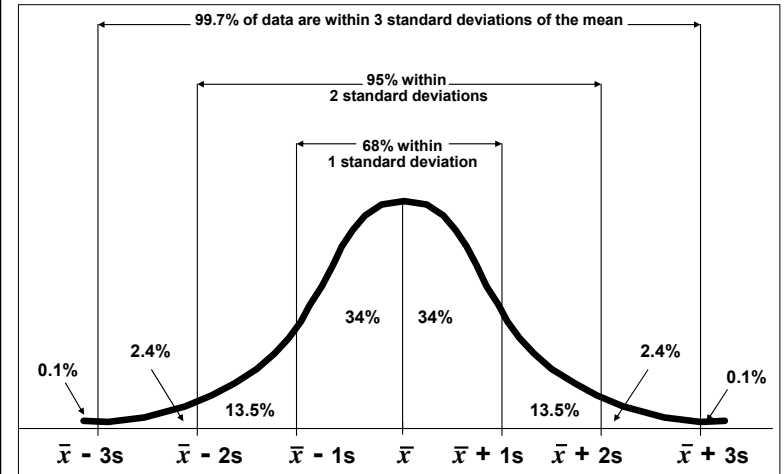
The Empirical Rule (applies to bell-shaped distributions)

FIGURE 2-13



The Empirical Rule (applies to bell-shaped distributions)

FIGURE 2-13



Measures of Variation Summary

For typical data sets, it is unusual for a score to differ from the mean by more than 2 or 3 standard deviations.