3 - 4

Multiplication Rule: Basics

Finding the Probability of Two or More Selections

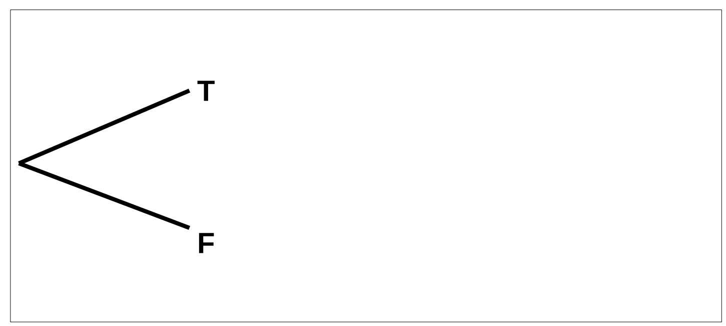
- Multiple selections
- Multiplication Rule

Notation

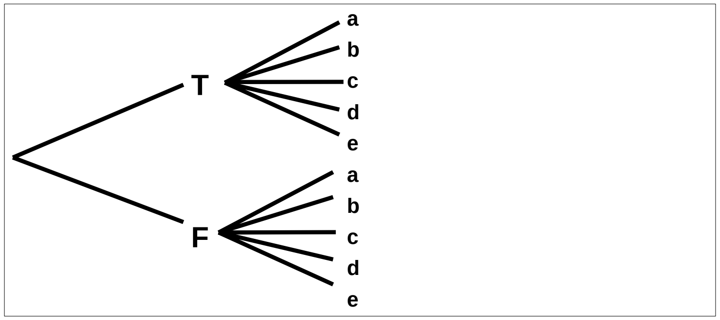
P(A and B) =

P(event A occurs in a first trial and event B occurs in a second trial)

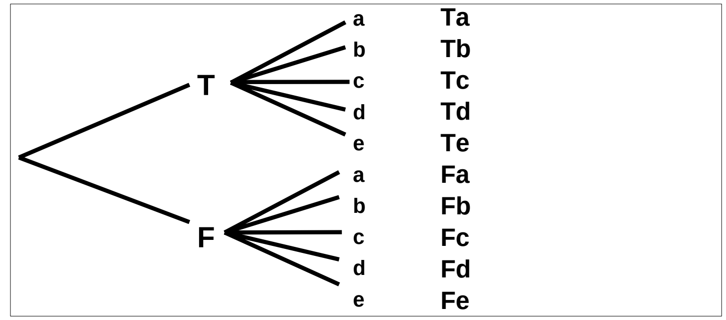
Tree Diagram of Test Answers



Tree Diagram of Test Answers



Tree Diagram of Test Answers



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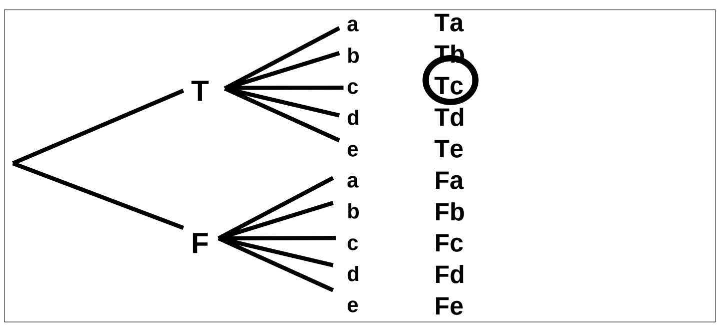
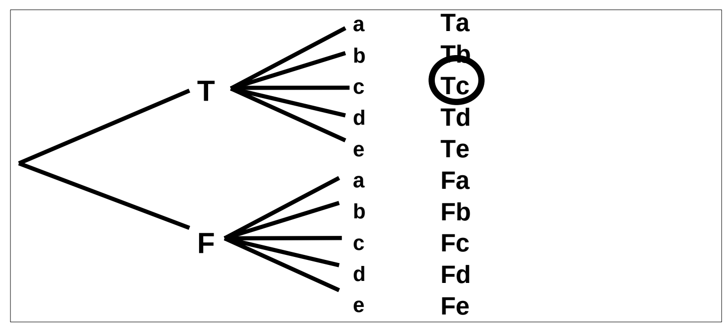


Tree Diagram of Test Answers



$P(T) = \frac{1}{2}$ $P(c) = \frac{1}{5}$ $P(T \text{ and } c) = \frac{1}{10}$

P (both correct) = P (T and c)

$$\frac{1}{10} = \frac{1}{2} \cdot \frac{1}{5}$$

Multiplication Rule

P (both correct) = P (T and c)

$$\frac{1}{10} = \frac{1}{2} \cdot \frac{1}{5}$$

Multiplication Rule

INDEPENDENT EVENTS

Notation for Conditional Probability

P(B|A) represents the probability of event B occurring after it is assumed that event A has already occurred (read B|A as "B given A").

Definitions

- **Independent Events**

  Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other.

- **Dependent Events**

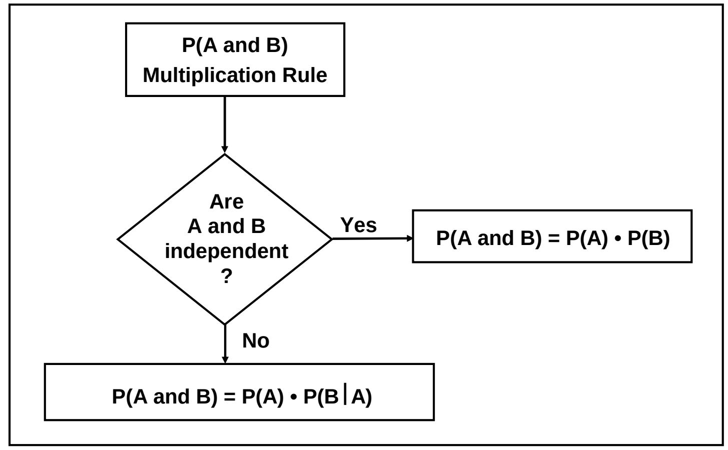
  If A and B are not independent, they are said to be dependent.

Formal Multiplication Rule

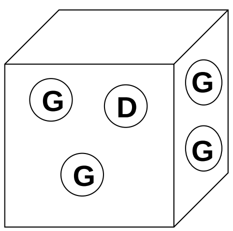
❖P(A and B) = P(A) • P(B|A)

❖If A and B are independent events, P(B|A) is really the same as P(B)

Applying the Multiplication Rule



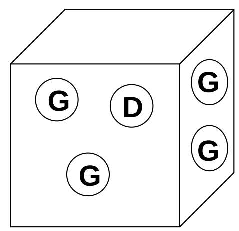
Independent Events



- Two selections
- <u>With</u> replacement

P (both good) =

Independent Events

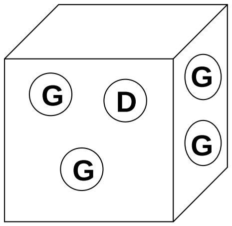


- Two selections
- <u>With</u> replacement

P (both good) =

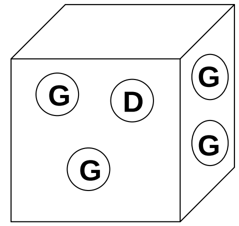
P (good <u>and</u> good) =

$$\frac{4}{5} \circ \frac{4}{5} = \frac{16}{25} = 0.64$$



- Two selections
- <u>Without</u> replacement

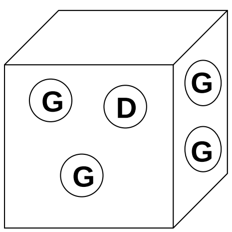
P (both good) =



- Two selections
- <u>Without</u> replacement

P (both good) =

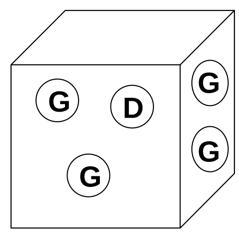
P (good) and P(good) =



- Two selections
- <u>Without</u> replacement

P (both good) =

P (good) ◦ P(good) =



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- <u>Without</u> replacement

P (both good) =

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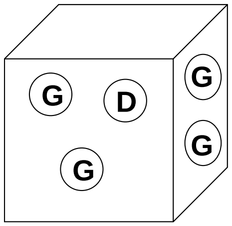
$\frac{4}{5}$



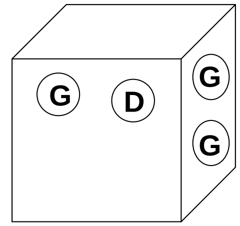
- Two selections
- **Without** replacement

P (both good) =

P (good) $\circ$ P(good) =

$\frac{4}{5}$

(assume good was selected)



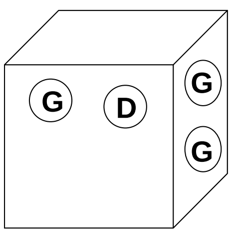
- Two selections
- **Without** replacement

P (both good) =

P (good) $\circ$ P(good) =

$\frac{4}{5} \circ \frac{3}{4}$ ← 3 good left ← 4 dryers left

(assume good was selected)



- Two selections
- **Without** replacement

P (both good) =

P (good) $\circ$ P(good) =

$\frac{4}{5} \circ \frac{3}{4}$ ← 3 good left ← 4 dryers left $= \frac{12}{20} = 0.60$

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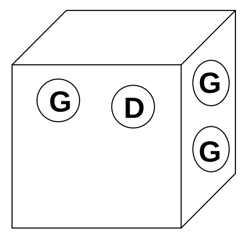
DEPENDENT EVENTS



- Two selections
- **Without** replacement

P (both good) =

P (good) $\circ$ P(good) =

$\frac{4}{5} \circ \frac{3}{4}$ ← 3 good left ← 4 dryers left $= \frac{12}{20} = 0.60$

(assume good was selected)

Intuitive Multiplication

When finding the probability that event A occurs in one trial and B occurs in the next trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B takes into account the previous occurrence of event A.

Find the probability of drawing two cards from a shuffled deck of cards such that the first is an Ace and the second is a King. (The cards are drawn <u>without replacement</u>.)

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- P(drawing Ace, then a King) = $\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652}$

  $= 0.00603$

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DEPENDENT EVENTS

Example: On a TV program it was reported that there is a 60% success rate for those who try to stop smoking through hypnosis. Find the probability that for 8 randomly selected smokers who undergo hypnosis, they all successfully quit smoking.

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P(all 8 quit smoking) =

P(quit) P(quit) P(quit) P(quit) P(quit) P(quit) P(quit) P(quit) =

$(0.60) \bullet (0.60) \bullet (0.60) \bullet (0.60) \bullet (0.60) \bullet (0.60) \bullet (0.60) \bullet (0.60)$

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(0.60) • (0.60) • (0.60) • (0.60) • (0.60) • (0.60) • (0.60) • (0.60)

or

$0.60^8 = 0.0168$

Small Samples from Large Populations

If a sample size is no more than 5% of the size of the population, treat the selections as being *independent* (even if the selections are made without replacement, so they are technically dependent).

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INDEPENDENT

$$\left(\frac{43650}{970000}\right)^4 =$$

0.000004100625

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INDEPENDENT

$(0.045)^4 =$

0.000004100625

DEPENDENT

$$\frac{43650}{970000} \cdot \frac{43649}{969999} \cdot \frac{43648}{969998} \cdot \frac{43647}{969997} =$$

0.000004100086725

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INDEPENDENT

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DEPENDENT

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0.000004100086725 ↑

Independence

You must treat a problem as independent when:

- you do not have the sample or population size, and
- you have only a percentage (probability) of the individual characteristic