Show all work and answers in order on your own paper. (All problems are 9 points each unless otherwise listed)

1. Graph the following linear inequality: \(3x + 4y \geq 12\) (5 pts)

2. Graph the feasible region for the systems of inequalities:
   \[
   \begin{align*}
   x + y &\leq 6 \\
   2x - y &\geq 3
   \end{align*}
   \]

3. Determine the constraints that would yield a feasible region of the interior of a triangle with vertices (-2,4), (5,8), and (6,4). (8 pts)

For problems 4 and 5, use graphical methods to find the feasible region. Optimize the solution by evaluating the corner points.

4. Maximize: \(z = 10x + 12y\)
   \[
   \begin{align*}
   3x + y &\leq 15 \\
   x + 2y &\leq 18 \\
   x &\geq 0, y &\geq 0
   \end{align*}
   \]

5. Minimize: \(z = 4x + 5y\)
   \[
   \begin{align*}
   10x - 5y &\leq 10 \\
   20x + 10y &\geq 150 \\
   x &\geq 0, y &\geq 0
   \end{align*}
   \]

Solve the following application graphically. Show what \(x\) and \(y\) should represent.

6. Kim has a nutritional deficiency and is told to take at least 2400mg of iron, 2100 mg of vitamin B-1, and 1500 mg of vitamin B-2. One herbal supplement contains 40 mg of iron, 10 mg of B-1, and 5 mg of B-2 and costs $.06 per pill. Another pill provides 10 mg of iron, 15 mg of B-1, and 15 mg of B-2 and costs $.08. What combination of pills will meet the requirements at the lowest cost? What is the minimum cost?

Solve 7,8 using the Simplex Method. Show all simplex tableaus needed for solution and circle all pivot points. Show solutions for all variables and objective function.

7. Maximize: \(z = 5x_1 + x_2\)
   \[
   \begin{align*}
   2x_1 + 3x_2 &\leq 8 \\
   4x_1 + 8x_2 &\leq 12 \\
   5x_1 + 2x_2 &\leq 30 \\
   x_1 &\geq 0, x_2 &\geq 0
   \end{align*}
   \]

8. Maximize: \(z = 3x_1 + 2x_2 - 4x_3\)
   \[
   \begin{align*}
   x_1 - x_2 + x_3 &\leq 10 \\
   2x_1 - x_2 + 2x_3 &\leq 30 \\
   -3x_1 + x_2 + 3x_3 &\leq 40 \\
   x_1 &\geq 0, x_2 &\geq 0, x_3 &\geq 0
   \end{align*}
   \]

Solve the maximization word problem for #9. Make sure you include a written solution containing answers for all variables (and slack variables) and your objective function.
9. Patio Iron makes wrought iron outdoor dining tables, chairs, and stools. Each table uses 8 feet of standard width wrought iron, 4 hours of labor for cutting and assembly, and 2 hours of labor for detail and finishing work. Each chair uses 6 feet of the wrought iron, 2 hours of cutting and assembly labor, and 1.5 hours of detail and finishing labor. Each stool uses 1 foot of wrought iron, 1.5 hours for cutting and assembly, and .5 hours for detail and finishing work, and the daily demand for stools is at most 16. Each day Patio Iron has available at most 108 feet of wrought iron, 50 hours for cutting and assembly, and 40 hours for detail and finishing work. If the profits are $60 for each dining table, $48 for each chair, and $36 for each stool, how many of each item should be made each day to maximize profit? Find the maximum profit? Is there anything left over after achieving the maximum profit? (15 pts)

Use the Simplex Method Duality Property to minimize the following problems:

10. Minimize \( w = 11y_1 + 7y_2 \)
    subject to: \( y_1 + 2y_2 \geq 10 \)
                \( 3y_1 + y_2 \geq 15 \)

11. Minimize \( w = 18y_1 + 12y_2 \)
    subject to: \( 2y_1 + y_2 \geq 8 \)
                \( 6y_1 + 6y_2 \geq 36 \)