INTEGRALS

5.8 Integration Using Tables

Objective: Evaluate integrals using tables of integrals

I. It is frequently necessary to use substitution along with integral tables

II. It may be necessary to use a formula more than once or to use two or more formulas

III. Evaluate $\int_0^2 \left(\frac{x^2 + 12}{x^2 + 4} \right) dx$

A. The closest formula is # 17: $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

B. Divide: $\frac{x^2 + 12}{x^2 + 4} = 1 + \frac{8}{x^2 + 4}$

C. Applying #17 with a = 2: $\int_0^2 \left(\frac{x^2 + 12}{x^2 + 4} \right) dx = \int_0^2 \left(1 + \frac{8}{x^2 + 4} \right) dx$

$$= \left[x + 8 \cdot \frac{1}{2} \tan^{-1} \frac{x}{2} \right]_0^2 = 2 + 4\tan^{-1} 1 = 2 + p$$

IV. Evaluate $\int \frac{x^2}{\sqrt{5 - 4x^2}} dx$

A. The closest formula is #34: $\int \frac{u^2}{\sqrt{a^2 - u^2}} du = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{u}{a} \right) + C$

B. Using substitution: let u = 2x $\Rightarrow du = 2dx$ and $x = \frac{u}{2}$

$$\int \frac{x^2}{\sqrt{5 - 4x^2}} dx = \frac{1}{2} \int \frac{\left(\frac{u}{2} \right)^2}{\sqrt{5 - u^2}} du$$

C. Using # 34 with $a^2 = 5$: $\frac{1}{2} \int \frac{\left(\frac{u}{2} \right)^2}{\sqrt{5 - u^2}} du = \frac{1}{8} \int \frac{u^2}{\sqrt{5 - u^2}} du$

$$= \frac{1}{8} \left[-\frac{u}{2} \sqrt{5 - u^2} + \frac{5}{2} \sin^{-1} \frac{u}{\sqrt{5}} \right] + C$$

$$= \frac{1}{8} \left[-\frac{u}{2} \sqrt{5 - u^2} + \frac{5}{2} \sin^{-1} \frac{u}{\sqrt{5}} \right] + C$$

V. Evaluate $\int x^3 \sin x dx$

A. Using #84 with n = 3: $\int x^3 \sin x dx = -x^3 \cos x + 3\int x^2 \cos x dx$

B. Using #85 with n = 2: $\int x^2 \cos x dx = x^2 \sin x - 2\int x \sin x dx$

C. Using #82: $\int x \sin x dx = \sin x - x \cos x$

D. Combining all results: $\int x^3 \sin x dx = -x^3 \cos x + 3x^2 \sin x - 6 \sin x + 6x \cos x + C$