APPLICATIONS OF INTEGRATION

6.1 More About Areas

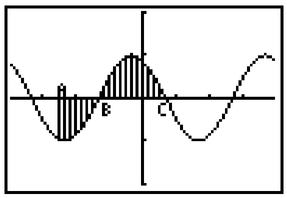
Objective: Find the area between two curves

I. The area between the graph of a continuous curve on [a, b] and the x-axis

A. If $f(x) \geq 0$ on [a, b], then $A = \int_a^b f(x)dx$

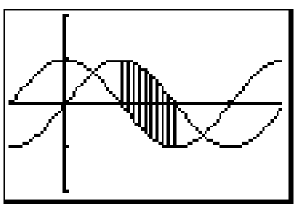
B. If $f(x) \leq 0$ on [a, b], then $A = \left|\int_a^b f(x)dx\right| = -\int_a^b f(x)dx = \int_b^a f(x)dx$

II. Find the shaded area:



$A = \left|\int_a^b f(x)dx\right| + \int_b^c f(x)dx$

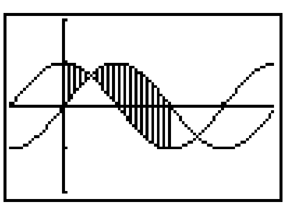
III. Find the area between the graphs of y = sin(x) and y = cos(x) on $\left[\frac{p}{2}, p\right]$



$$A = \int_{p/2}^{p} (\sin x)dx - \int_{p/2}^{p} \cos(x)dx = \int_{p/2}^{p} [\sin(x) - \cos(x)]dx$$

$$= [-\cos(x) + \sin(x)]_{p/2}^{p} = -\cos(p) + \sin(p) - \left[-\cos\left(\frac{p}{2}\right) + \sin\left(\frac{p}{2}\right)\right] = 2$$

IV. Find the area between the graphs of y = sin(x) and y = cos(x) on $[0, p]$



A. The area between the graph of y = sin(x) and the x-axis is $A_1 = \int_0^p (\sin x)dx$

B. The area between the graph of y = cos(x) and the x-axis is

$$A_2 = \int_0^{p/2} \cos(x)dx - \int_{p/2}^{p} \cos(x)dx$$

C. The only point where sin(x) = cos(x) on $[0, p]$ is at $x = \frac{p}{4}$

D. The shaded area is

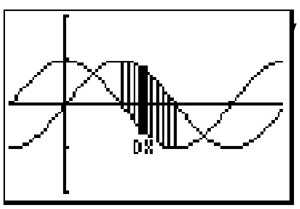
$$A = \int_0^{p/4} \cos(x)dx - \int_0^{p/4} \sin(x)dx + \int_{p/4}^{p} (\sin x)dx - \int_{p/4}^{p/2} \cos(x)dx - \int_{p/2}^{p} \cos(x)dx$$

$$= \frac{\sqrt{2}}{2} - \left(1 - \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2} + 1\right) - \left(1 - \frac{\sqrt{2}}{2}\right) - (-1) = 2\sqrt{2}$$

$$= \int_0^{p/4} [\cos(x) - \sin(x)]dx + \int_{p/4}^{p} [\sin(x) - \cos(x)]dx = (\sqrt{2} - 1) + (\sqrt{2} + 1) = 2\sqrt{2}$$

E. If f(x) and g(x) are continuous and $f(x) \geq g(x)$ on [a, b], then the area bounded by the graphs of f(x), g(x), x = a, and x = b is $\int_a^b [f(x) - g(x)]dx$ "top" – "bottom"

V. To find the area between the graphs of two continuous functions f(x) and g(x) on [a, b], where $f(x) \geq g(x)$, draw a typical rectangle, and label its width and height

A. 

= cos(x)

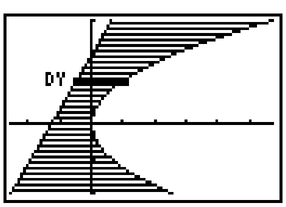
y = sin(x)

Its width is (dx) and its height is [f(x) - g(x)].

Its area is $\int_a^b [f(x) - g(x)]dx$

B. The area between y = cos(x) and y = sin(x) on $\left[\frac{p}{2}, p\right]$ is $\int_{p/2}^{p} [\sin(x) - \cos(x)]dx = 2$

VI. Find the area between the graphs of x = y^2 and $y - x = 2$ on $-2 \leq y \leq 3$

A. 

f(y) = y^2

g(y) = y - 2

Its width is (dy) and its height is [f(y) - g(y)].

Its area is $\int_c^d [f(y) - g(y)]dy$

B. The area between $x = y^2$ and $x = y - x$ is

$$\int_{-2}^{3} [y^2 - (y - 2)]dy = \int_{-2}^{3} (y^2 - y + 2)dy = \frac{115}{6}$$

C. If f(y) and g(y) are continuous and $f(y) \geq g(y)$ on [c, d], then the area bounded by the graphs of f(y), g(y), y = c, and y = d is $\int_c^d [f(y) - g(y)]dy$ "right" – "left"