APPLICATIONS OF INTEGRATION

6.2 Volumes

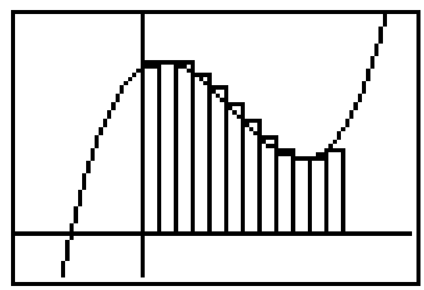
Objective: Find volume of solids of revolution

I. Definition of volume

A. S is a solid which lies between x = a and x = b
B. Cross-sectional area of S in plane P_x, through x and perpendicular to x-axis is A(x)
C. A is a continuous function
D. $V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)dx$

II. A ***solid of revolution*** is formed by revolving a region in a plane about a line in the plane, called the axis of revolution.

III. ***Disk method***: Find the volume of the solid of revolution generated by revolving the region formed by $y = \frac{1}{2}x^3 - 2x^2 + x + 4$, x = 0, x = 3, and the x-axis about the x–axis.



A. Consider a regular partition on [0, 3] with $\Delta x = \frac{1}{4}$
B. Rotate area about x–axis
C. Each rectangle generates a circular disk
D. Volume of a disk = $p r^2 h$
E. Radius of a disk = f (x)
F. Height (thickness) of a disk = Δx
G. Volume of a disk = $p\,(\text{radius})^2\,(\text{thickness})$
H. Volume of a disk = $p\left(\frac{1}{2}x^3 - 2x^2 + x + 4\right)^2 \Delta x$

I. $$V_L = p\left[f(0)^2 + f\left(\frac{1}{4}\right)^2 + f\left(\frac{2}{4}\right)^2 + f\left(\frac{3}{4}\right)^2 + f\left(\frac{4}{4}\right)^2 + f\left(\frac{5}{4}\right)^2 + f\left(\frac{6}{4}\right)^2 + f\left(\frac{7}{4}\right)^2 + f\left(\frac{8}{4}\right)^2 + f\left(\frac{9}{4}\right)^2 + f\left(\frac{10}{4}\right)^2 + f\left(\frac{11}{4}\right)^2\right]\left(\frac{1}{4}\right)$$

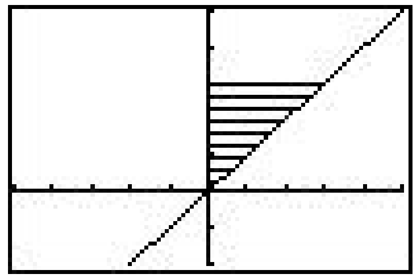
$V_L \approx 28.35p$, $V_R \approx 25.91p$, $V_M \approx 27.10p$

J. $V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)dx = \int_a^b p\,[f(x)]^2 dx = \int_0^3 p \left(\frac{1}{2}x^3 - 2x^2 + x + 4 \right)^2 dx$

$= \int_0^3 p \left(\frac{x^6}{4} - 2x^5 + 5x^4 - 15x^2 + 8x + 16 \right) dx$

$= p \left[\frac{x^7}{28} - \frac{x^6}{3} + x^5 - 5x^3 + 4x^2 + 16x \right]_0^3 = \frac{759}{28} p$

IV. Find the volume of the solid of revolution formed by rotating the region bounded by y = x, y = 3, and x = 0 about the y–axis.



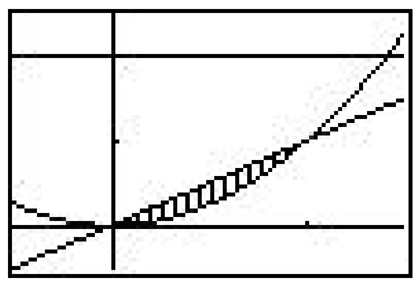
A. Thickness = Δy

B. Radius = f(y) = y

C. Volume of a disk = $p\,y^2\Delta y$

D. $V = \int_0^3 p\,y^2 dy = p\left[\frac{y^3}{3}\right]_0^3 = 9p$

V. Washer method: Find volume of solid obtained by rotating the region enclosed by the curves y = x and y = x^2 about the line y = 2



A. Thickness = Δx

B. Cross-section is an annulus [ring or washer]

C. Outer radius = $2 - x^2$; inner radius = $2 - x$

D. Volume of a washer = $[p(2 - x^2)^2 - p(2 - x)^2]\Delta x$

E. $V = \int_0^1 p[(2 - x^2)^2 - (2 - x)^2]dx = p\int_0^1 (x^4 - 5x^2 + 4x)dx = \frac{8p}{15}$

VI. Summation of cross-sections may also be used to find volumes of solids which are not revolved about an axis: see Example 6 on p. 460