APPLICATIONS OF INTEGRATION

6.4 Average Value of a Function

Objective: Compute the average value of a function

I. Average value of f on the interval [a, b]

A. $f_{ave} = \dfrac{1}{b-a}\int_a^b f(x)dx$

B. Find the average value of $f(x) = x^2$ on [0, 2]

$$f_{ave} = \frac{1}{2}\int_0^2 x^2 dx = \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{4}{3}$$

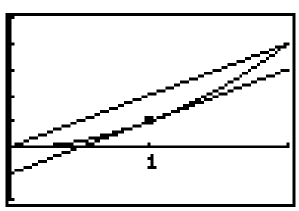
II. Mean Value Theorem for derivatives

If f is a differentiable function on the interval [a, b], then there exists a number c between a and b such that $f(b) - f(a) = f'(c)(b-a)$

A. $f(x) = x^2$ is continuous on [0, 2] and $f'(x) = 2x$

B. There exists a number c in [0, 2] such that $f(2) - f(0) = 2c(2 - 0)$

$$4 - 0 = 4c \Rightarrow c = 1$$

C. 

0 2

D. If f is differentiable on [a, b], then there is a number c between a and b such that the tangent at c is parallel to the secant AB

III. Mean Value Theorem for integrals

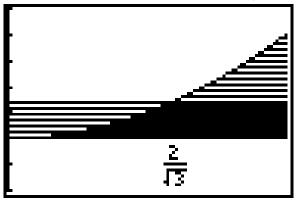
If f is continuous on [a, b], then there exists a number c in [a, b] such that

$$\int_a^b f(x)dx = f(c)(b-a)$$

1. $f(c) = f_{ave} = \dfrac{1}{2}\int_0^2 x^2 dx = \dfrac{4}{3}$

2. $c^2 = \frac{4}{3} \Rightarrow c = \frac{2}{\sqrt{3}}$

3.



4. If f is a positive function on [a, b], then there is a number c such that the rectangle with base [a, b] and height f(c) has the same area as the region under the graph of f from a to b.