**Pellissippi State**

**2010**

*Middle School Mathematics Competition*

Sponsored by: Oak Ridge Associated Universities

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**Eighth Grade**

Scoring Formula: 4R – W + 32

**Directions:**

For each problem there are 5 possible answers listed. You are to work the problems, determine the correct answer, and indicate your choice on the separate answer sheet provided.

Please use only capital letters on the answer sheet (A, B, C, D, E) and print neatly. This will more easily enable us to correctly grade your paper. If there is any question as to what letter an answer is, it will be marked wrong.

If you change your mind about your answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. Additional room for you to work out problems is available on the back of each of the test booklet’s pages.

When told to do so, open your test booklet and begin. When you have finished one page, go on to the next. There are 32 questions in all. The working time for the entire test is 60 minutes.
1. The graph below shows the total distance a family traveled over time.

![Graph showing distance traveled over time]

The family traveled a total distance of 500 miles. What percentage of the total trip had they traveled by the end of the third hour?

A. 24%  
B. 20%  
C. 50%  
D. 33%  
E. 28%

2. Which of the following is not equivalent to 0.0025% of 40,000?

A. $0.0025 \times 40,000$  
B. $\frac{1}{400}$ of 40,000  
C. 1% of 100  
D. $2.5\% \times 40$  
E. 5% of 20

3. Jason, Mike, and Frank took turns driving from Houston to Miami. First, Jason drove $\frac{3}{11}$ of the total distance. Next, Mike drive 25% of the total distance. Finally, Frank drove the remaining 567 miles. What is the total driving distance between Houston and Miami?

A. 1,104 miles  
B. 1,205 miles  
C. 1,188 miles  
D. 1,473 miles  
E. 1,456 miles

4. John drew a geometric figure on a sheet of typing paper (8.5 inches by 11 inches). The typing paper weighs about 15.3 grams. He cut out the figure he drew. The weight of the figure was four grams. What is the area of the geometric figure he cut out?

A. 6 square inches  
B. 0.2614 square inches  
C. 61.2 square inches  
D. $24\frac{4}{9}$ square inches  
E. $12\frac{1}{3}$ square inches
5. An apple has 80 calories. A banana has 37.5% more calories than an apple. A banana has 21.5% of the calories in a hamburger. How many calories are in a hamburger? (Round to the nearest calorie.)

A. 512 calories  
B. 527 calories  
C. 548 calories  
D. 364 calories  
E. 359 calories

6. The first four figures of a sequence are shown below. How many dots would be in the 50th figure in the sequence?

\[
\begin{array}{ccccccc}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

A. 2452  
B. 2450  
C. 2448  
D. 2654  
E. 2655

7. The product, \( p \), of two prime numbers is between 6 and 55. The first prime is greater than 2 and less than 6. The second prime is greater than 13 and less than 25. What is \( p \)?

A. 26  
B. 31  
C. 33  
D. 41  
E. 51

8. In 2007, Sal's salary for the year was $23,474. Each year Sal's yearly salary increases by the same amount. In 2015, Sal's yearly salary will be $54,210. What is Sal's yearly salary for 2010?

A. $34,000  
B. $35,000  
C. $36,000  
D. $41,000  
E. $43,000

9. From a batch of 3,000 light bulbs, 100 were selected at random and tested. Five of the light bulbs in the sample were found to be defective. About how many defective light bulbs would be expected in the entire batch?

A. 600  
B. 15  
C. 300  
D. 60  
E. 150
10. The population density for a country is the number of people per square mile. Monaco’s population is $3.3 \times 10^4$ people. The area of Monaco is 0.75 square miles. What is the population density of Monaco?

A. 4,400 people per square mile
B. 440 people per square mile
C. 44,000 people per square mile
D. 440,000 people per square mile
E. 0.44 people per square mile

11. Which of the following statements is FALSE? (Note: an $n$-gon is a polygon with $n$ sides.)

A. The number of edges of a pyramid whose base is an $n$-gon is $2n$.
B. The number of vertices of a pyramid whose base is an $n$-gon is $n + 1$.
C. The number of faces of a prism whose bases are $n$-gons is $n + 4$.
D. The number of vertices of a prism whose bases are $n$-gons is $2n$.
E. The number of edges of a prism whose bases are $n$-gons is $3n$.

12. The Concorde passenger plane weighs $4.08 \times 10^5$ pounds, and a cricket weighs $3.125 \times 10^{-4}$ pounds. How many crickets are needed to weigh the same as a Concorde passenger plane?

A. $1.3056 \times 10^8$
B. $1.3056$
C. $1.3056 \times 10^1$
D. $1.3056 \times 10^{10}$
E. $1.3056 \times 10^9$

13. A survey asked 750 people about sports. The survey found that 68% liked football, 54% liked basketball, and 18% liked neither football nor basketball. How many people liked both football and basketball?

A. 165
B. 300
C. 135
D. 40
E. 615

14. A common foodstuff is found to contain 0.00125% iron. The serving size is 87.0 grams. If the recommended daily allowance is 18 milligrams of iron, how many servings would a person have to eat to get 100% of the daily allowance of iron? Round your answer to the nearest whole serving.

A. 5 servings
B. 17 servings
C. 1 servings
D. 16,552 servings
E. 920 servings

15. There will be five tests in your history class. You have scores of 62, 76, and 90 on the first three tests. What should be the average of your scores on Test 4 and Test 5 so that the mean of all five tests is exactly 80?

A. 82
B. 83
C. 84
D. 85
E. 86
16. Which of the following situations is reasonable?

   A. A two-year old is 30 meters tall.
   B. A large suitcase has a volume of 4 cubic decimeters.
   C. A pair of shoes weighs 0.5 kilograms.
   D. The area of the gym floor is 400 square decimeters.
   E. A dining table is 30 centimeters tall.

17. Which of the following statements is true?

   A. Twenty is 150% more than 8.
   B. Twenty is 150% of 8.
   C. Eight is 150% less than 20.
   D. Eight is 150% of 20.
   E. Eight is 50% less than 20.

18. Which of the following numbers is a rational number?

   A. \( \sqrt[3]{\frac{64}{121}} \)
   B. \( \sqrt{\frac{16}{35}} \)
   C. \( \sqrt{4\pi^2} \)
   D. \( \sqrt{\frac{23 - 26}{121}} \)
   E. \( \sqrt{54} \)

19. The table below shows the total values of various combinations of coins. (Assume all coins are in the same condition.) For example, the table that shows that two 1926 wheat pennies and two 1923 quarters would have a total combined value of $36.00.

<table>
<thead>
<tr>
<th>Number of 1926-S Wheat Pennies</th>
<th>Number of 1913 Buffalo Nickels</th>
<th>Number of 1923 Standing Liberty Quarters</th>
<th>Collection Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>$36.00</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>$28.50</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td>$54.00</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$22.50</td>
</tr>
</tbody>
</table>

What is the value of a 1913 buffalo nickel?

A. $4.00  B. $4.50  C. $2.50
D. $15.00  E. $3.00
20. Which of the five expressions given below is not equivalent to the other four?

A. \(50 \cdot 2^{-x}\)  
B. \(50 \cdot 0.5^x\)  
C. \(\frac{50}{2^x}\)

D. \(\frac{50}{0.5^{-x}}\)  
E. \(100^{-x}\)

21. Suppose that the regular hexagon shown below has a value of 1. In what base would the shaded area be represented by 0.3.

![Hexagon Diagram]

A. Base six  
B. Base twelve  
C. Base eighteen  
D. Base three  
E. Base two

22. The graph below represents a function that models the amount of a radioactive substance over time.

![Graph]

The number of grams of the radioactive substance is represented on the vertical \(y\)-axis. The number of years is given on the horizontal \(x\)-axis. The half-life is the amount of time required for half the substance to decay to a non-radioactive form. According to the graph, what is the half-life of this substance?

A. 6 years  
B. 1500 years  
C. 750 years  
D. 12 years  
E. 6000 years
23. Alden asked for her birthday cake to be a rectangular prism. The dimensions of the cake (without icing) are 10 inches by 15 inches by 2 inches. Icing is spread on the sides and top of the cake but not on the bottom of the cake. The icing is one-half inch thick. The cake was cut into 20 equal-size as shown below. How much more icing is there on a corner piece than on one cut from the middle?

A. 5.5 cubic inches  
B. 11 cubic inches  
C. 2.75 cubic inches  
D. 7.5 cubic inches  
E. 11.5 cubic inches

24. A spring scale weighs to the nearest half pound. A fisherman caught 6 fish and weighed each one individually with this scale. He added those weights together. Then he decided to put all the fish on the scale at the same time. What is the largest possible difference between sum of the individual weights and the scale reading when all fish were weighed at the same time?

A. The two amounts will be the same because the scale weighs to the nearest half pound.  
B. The two amounts could differ by a maximum of 0.25 pound.  
C. The two amounts could differ by a maximum of 0.5 pounds.  
D. The two amounts could differ by a maximum of 1.25 pounds.  
E. The two amounts could differ by a maximum of 1.5 pounds.

25. A right triangle is inscribed in a circle. (All the vertices of the triangle lie on the circle.) The coordinates of its vertices are given below.

A = (3, 1)  
B = (9, 1)  
C = (9, -7)

How long is the diameter of the circle? Let 1 unit = the distance between (0, 0) and (0, 1).

A. The diameter is 6 units long.  
B. The diameter is 12 units long.  
C. The diameter is 8 units long.  
D. The diameter is $12\sqrt{2}$ units long.  
E. The diameter is 10 units long.
26. The graphs of two functions are given below. What are the equations for those two functions?

\[ y = f(x) \]

\[ y = g(x) \]

A. \( f(x) = 4 \cdot (1.2)^x \) and \( g(x) = -2x + 4 \)
B. \( f(x) = 4 \cdot (1.2)^x \) and \( g(x) = 4 \cdot 3^x \)
C. \( f(x) = 4 \cdot (1.2)^x \) and \( g(x) = 0.25(x + 4)^2 \)
D. \( f(x) = -2x + 4 \) and \( g(x) = 0.25(x + 4)^2 \)
E. \( f(x) = 4 \cdot 3^x \) and \( g(x) = 0.25(x + 4)^2 \)

27. In the figure below, the measurements of the various segments are given.

\[ \overline{AB} = 9.54 \text{ cm} \]
\[ \overline{DA} = 5.00 \text{ cm} \]
\[ \overline{DB} = 5.00 \text{ cm} \]
\[ \overline{DC} = 5.00 \text{ cm} \]
\[ \overline{CB} = 3.00 \text{ cm} \]

Which of the statements is FALSE?

A. The area of triangle ABD = the area of triangle DBC.
B. Triangle ABD is an isosceles triangle.
C. Triangle BDC is an acute triangle.
D. The area of triangle BCD is 7.5 square centimeters.
E. Triangle ABD is an obtuse triangle.
28. In modular arithmetic, the statement $15 \equiv 3 \pmod{4}$ means that 15 and 3 both have the same remainder when they are divided by 4. The following are true statements in modular arithmetic:

\[
12 \equiv 7 \pmod{5} \\
7 \equiv 13 \pmod{2} \\
14 \equiv 42 \pmod{7}
\]

Which of the following statements is **FALSE**?

A. $12^2 \equiv 7^2 \pmod{5}$  
B. $12 \cdot 2 \equiv 7 \cdot 2 \pmod{5}$  
C. $12 + 4 \equiv 7 + 4 \pmod{5}$  
D. $12 + 10 \equiv 7 + 25 \pmod{5}$  
E. $12 \times 7 \equiv 0 \pmod{5}$

29. Multiplication of whole numbers can be defined as a function ($f$) from the set of all ordered pairs of whole numbers ($a, b$) onto the set of all whole numbers. Here are some examples:

\[
\begin{align*}
  f(3, 5) &= 15 \\
  f(1, 8) &= 8 \\
  f(0, 7) &= 0
\end{align*}
\]

Which of the following equations is true?

A. $f(2, 9) = 11$  
B. $f(0, 0) = 0$  
C. $f(0, 1) = 1$  
D. $f(3, 3) = 3$  
E. $f(1, 1) = 0$

30. The inverse of a function “undoes” the function. For example, if $f(x) = x - 6$, then the inverse of $f(x)$ would be $g(x) = x + 6$. [Note that $f(10) = 4$ and $g(4) = 10$. In every case, if $f(a) = b$, then $g(b) = a$.] Which of the following functions would be the inverse of $h(x) = \frac{5x - 4}{3}$?

A. $p(x) = \frac{3}{5x - 4}$  
B. $q(x) = \frac{3}{5x + 4}$  
C. $r(x) = \frac{3x - 4}{5}$  
D. $s(x) = \frac{4 - 3x}{5}$  
E. $t(x) = \frac{3x + 4}{5}$

31. If Leah is 6 years older than Sue, and John is 5 years older than Leah, and the total of their ages is 41. Then how old is Sue?

A. 8  
B. 10  
C. 14  
D. 19  
E. 21

32. Jim is able to sell a hand-carved statue for $670 which was a 35% profit over his cost. How much did the statue originally cost him?

A. $496.30  
B. $512.40  
C. $555.40  
D. $574.90  
E. $588.20