

TMATYC
CALCULUS B TEST
Fall 2010

1. Determine the rate of change of y with respect to x for the function $y = (\sin x)^{3x^2}$

A. $3x^2 (\sin x)^{3x^2-1} (\cos x)$

B. $3x \cos x (\sin x)^{3x^2} \ln(\sec x)$

C. $3x (\sin x)^{3x^2} (e^{\sin x} + \ln(\cot x))$

D. $3x (\sin x)^{3x^2} (2 \ln(\sin x) + x \cot x)$

2. What are the first three non-zero terms in the Taylor Series expansion for the function

$$f(x) = x \cdot \ln(x)$$

centered at the value $x = 1$?

A. $\frac{(x-1)}{2} - \frac{(x-1)^2}{6} + \frac{(x-1)^3}{24}$

B. $(x-1)^2 + \frac{(x-1)^4}{4!} - \frac{(x-1)^6}{6!}$

C. $(x-1) + \frac{(x-1)^2}{2} - \frac{(x-1)^3}{6}$

D. $\frac{(x-1)^2}{2} - \frac{(x-1)^3}{6} + \frac{(x-1)^4}{24}$

3. Evaluate the limit $\lim_{h \rightarrow 0} \frac{\sqrt[3]{125+h} - 5}{h}$

A. $\frac{1}{25}$

B. $\frac{1}{75}$

C. $\frac{1}{625}$

D. $\frac{1}{925}$

4. Which x values in the interval $[2,6]$ produce a tangent line on the function $f(x) = x^3 - 9x$ which is parallel to the secant line across the interval's endpoints?

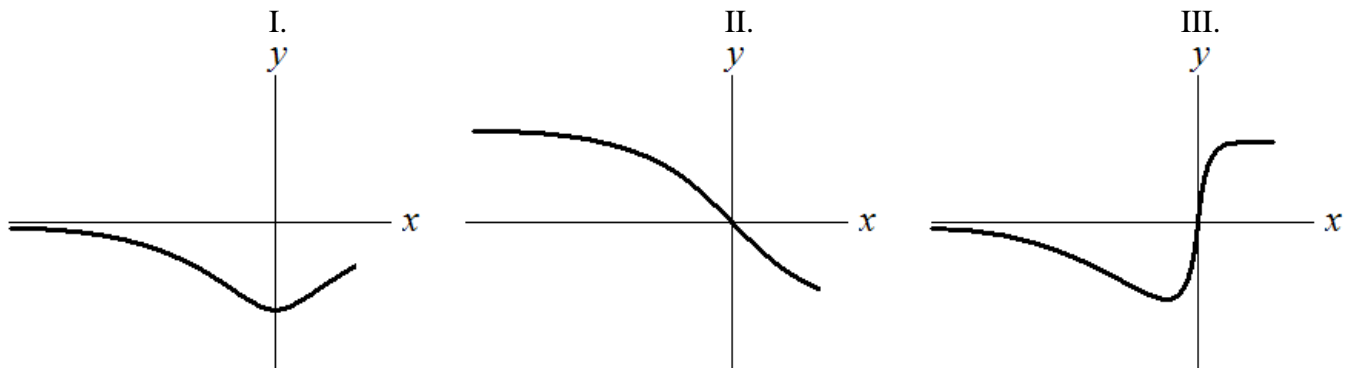
A. $x = 4$

B. $x = \frac{2}{3}\sqrt{39}$

C. $x = \sqrt{26}$

D. $x = \frac{5}{6}\sqrt{35}$

5. What is the domain of the derivative of the function $f(x) = x^{5/2}(x-2)^{2/3}$?
- A. \mathbb{R} B. $[2, \infty)$ C. $(0, 2) \cup (2, \infty)$ D. $[0, 2) \cup (2, \infty)$
6. What is the approximate maximum rate of change for $f(x, y) = 4 \sec x + 5y^3$ at the point $(\frac{\pi}{3}, -2)$?
- A. 61.6 B. 74.6 C. 89.3 D. 96.6
7. What is the derivative for the function $f(x) = \frac{2x}{\sec^2(20\pi x^5)} + \frac{2x}{\csc^2(20\pi x^5)}$?
- A. 2 B. $200\pi x(\tan^2(20\pi x^5) - \cot^2(20\pi x^5))$
 C. 0 D. $200\pi x$
8. Of the three graphs labeled I, II and III below, one is f , one is f' and one is f'' . Which of the following correctly identifies the three graphs?



- A. $f = \text{II}, f' = \text{III}, f'' = \text{I}$
 B. $f = \text{I}, f' = \text{II}, f'' = \text{III}$
 C. $f = \text{II}, f' = \text{I}, f'' = \text{III}$
 D. $f = \text{III}, f' = \text{I}, f'' = \text{II}$

9. What is the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{100^n(x-4)^n}{(3n)!}$?

- A. $(\frac{11}{3}, \frac{13}{3})$ B. $(-5, 13)$ C. $(-\infty, \infty)$ D. $\{4\}$

10. Integrate $\int \frac{14x^3}{x^2+1} dx$

A. $\frac{14}{3}(x^4 - \tan^{-1}(x)) + C$

B. $7x^2 - 7 + \tan^{-1}(x) + C$

C. $7x^2 - \ln(x^2 + 1) + C$

D. $7(x^2 - \ln(x^2 + 1)) + C$

11. Evaluate $\int_{-\sqrt{3}}^{\sqrt{3}} \tan^{-1}(3x) dx$

A. 6π

B. 0

C. $-\frac{2\pi}{3}$

D. $\frac{4\pi}{3}$

12. Evaluate the limit $\lim_{x \rightarrow 0^+} \left(1 + \frac{x}{3}\right)^{\frac{4}{5x}}$.

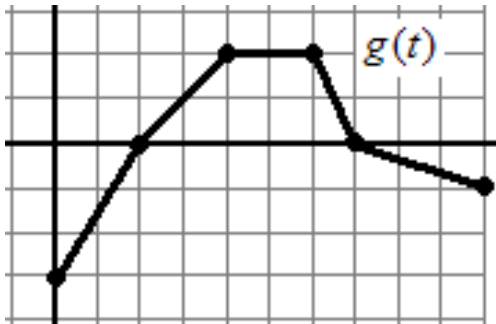
A. $\ln(0.2)$

B. $e^{4/15}$

C. $e^{1/3}$

D. ∞

13. The function shown below, $g(t)$, has a domain of $[0,10]$ and a range of $[-3,2]$.



The function $f(x)$ is defined as ...

$$f(x) = \int_0^x g(t) dt.$$

What is the minimum value of $f(x)$ on the interval $[0,10]$?

A. -3

B. 0

C. 1

D. -6

14. Solve the exact differential equation $(5y^2e^x) dx + (4 + 10ye^x) dy = 0$.

A. $8y^2 + 5e^x = C$

B. $4ye^x + 10y^2 = C$

C. $4y^2 + 10ye^x = C$

D. $4y + 5y^2e^x = C$

15. Evaluate the double integral $\int_0^{\sqrt{3\pi}} \int_0^x 5\sin(x^2) dy dx$.

A. 5

B. 10

C. 0

D. 6π

16. Determine the value of the area bounded between the graph of $y = \sin x$ and the graph of its tangent line at $x=0$ from the values $x=0$ to $x=\pi$.

A. $\frac{2}{3}(3\pi^2 - 1)$

B. $\frac{1}{2}(\pi^2 - 4)$

C. $\frac{1}{3}(\pi^2 - 2)$

D. $\frac{1}{4}(2\pi^2 + 1)$

17. Integrate $\int x \sec^2(7x) dx$

A. $\frac{1}{7} \sec 7x \tan 7x - \frac{1}{7} \ln |\sec 7x| + C$

B. $\frac{1}{7} x \tan 7x + \frac{1}{49} \ln |\cos 7x| + C$

C. $\frac{1}{7} \sec^2 7x + \frac{1}{7} \ln |\cos 7x| + C$

D. $\frac{1}{14} x^2 \tan 7x + \frac{1}{7} \ln |\cos 7x| + C$

18. Given the vector function $\mathbf{r}(t) = (4t - 3t^2)\mathbf{i} + (2\cos^{-1}t)\mathbf{j}$, evaluate the unit tangent vector $\mathbf{T}(0)$.

A. $-\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$

B. $\frac{2}{\sqrt{29}}\mathbf{i} + \frac{5}{\sqrt{29}}\mathbf{j}$

C. $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$

D. $-\frac{3}{\sqrt{10}}\mathbf{i} - \frac{1}{\sqrt{10}}\mathbf{j}$

19. For the first-order ODE $\sin(x)\frac{dy}{dx} + \cos(x)y = \sin(x)e^{\sec^2 x}$, determine the integrating factor $\mu(x)$.

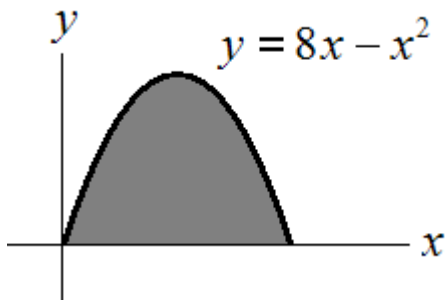
A. $e^{-\sin x}$

B. $|\sin(x)|$

C. $e^{\tan x}$

D. $|\tan x|$

20. Determine the volume of revolving the region shown below about the line $x = 12$.



A. $1024\pi/3$ cubic units

B. $4096\pi/3$ cubic units

C. $5132\pi/3$ cubic units

D. $6892\pi/3$ cubic units

21. A steady volume of sand is poured from a conveyor belt into a quarry below. The pile makes the shape of a right circular cone with a height equal to $2/3$ of the base diameter. At the instant the cone is 8 meters high, the height is growing at a rate of 0.6 m/min. What is the flow rate of the sand leaving the conveyor belt?

A. $48.7 \text{ m}^3/\text{min}$

B. $58.8 \text{ m}^3/\text{min}$

C. $67.9 \text{ m}^3/\text{min}$

D. $76.1 \text{ m}^3/\text{min}$

22. What is the area inside one petal of the three-leaf rose $r = 12 \cos(3\theta)$?

A. 12π

B. 6π

C. 8π

D. 18π

23. What is the area of the parallelogram spanned by the vectors $\mathbf{v} = 4\mathbf{i} - 7\mathbf{j}$ and $\mathbf{w} = -3\mathbf{i}$?

- A. $\sqrt{74}$ sq. units B. $\sqrt{53}$ sq. units C. 28 sq. units D. 21 sq. units

24. Integrate $\int \frac{8}{6t-t^2} dt$ is

- A. $\frac{2}{3} \ln |t^2 - 6t| + C$ B. $\frac{4}{3} \ln \left| \frac{t}{t-6} \right| + C$
C. $\frac{4}{3} \ln \left| \frac{t-6}{t} \right| + C$ D. $\frac{4}{3} (t - \tan^{-1}(3-t)) + C$

25. Evaluate the improper integral $\int_{-1}^1 \ln |2x| dx$.

- A. $4 + \ln 2$ B. $2 - \ln 8$ C. $-2 + \ln 4$ D. Divergent