TMATYC
CALCULUS B TEST
Fall 2010

1. Determine the rate of change of *y* with respect to *x* for the function $y = (\sin x)^{3x^2}$

   A. $3x^2(\sin x)^{3x^2-1}(\cos x)$
   B. $3x\cos x(\sin x)^{3x^2}\ln(\sec x)$
   C. $3x(\sin x)^{3x^2}\left(e^{\sin x}+\ln(\cot x)\right)$
   D. $3x(\sin x)^{3x^2}\left(2\ln(\sin x)+x\cot x\right)$

2. What are the first three non-zero terms in the Taylor Series expansion for the function

$$f(x) = x\cdot\ln(x)$$

   centered at the value *x* = 1 ?

   A. $\frac{(x-1)}{2}-\frac{(x-1)^2}{6}+\frac{(x-1)^3}{24}$
   B. $(x-1)^2+\frac{(x-1)^4}{4!}-\frac{(x-1)^6}{6!}$
   C. $(x-1)+\frac{(x-1)^2}{2}-\frac{(x-1)^3}{6}$
   D. $\frac{(x-1)^2}{2}-\frac{(x-1)^3}{6}+\frac{(x-1)^4}{24}$

3. Evaluate the limit $\lim_{h\to 0}\frac{\sqrt[3]{125+h}-5}{h}$

   A. $\frac{1}{25}$
   B. $\frac{1}{75}$
   C. $\frac{1}{625}$
   D. $\frac{1}{925}$

4. Which *x* values in the interval [2,6] produce a tangent line on the function $f(x) = x^3 - 9x$ which is parallel to the secant line across the interval's endpoints?

   A. $x = 4$
   B. $x = \frac{2}{3}\sqrt{39}$
   C. $x = \sqrt{26}$
   D. $x = \frac{5}{6}\sqrt{35}$

5. What is the domain of the derivative of the function $f(x) = x^{5/2}(x-2)^{2/3}$?

A. $\mathbb{R}$ B. $[2,\infty)$ C. $(0,2)\cup(2,\infty)$ D. $[0,2)\cup(2,\infty)$

6. What is the approximate maximum rate of change for $f(x,y) = 4\sec x + 5y^3$ at the point $(\frac{\pi}{3}, -2)$?

A. 61.6 B. 74.6 C. 89.3 D. 96.6

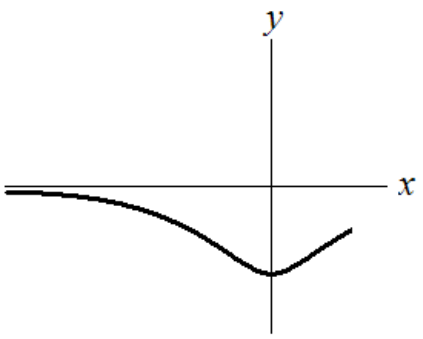
7. What is the derivative for the function $f(x) = \dfrac{2x}{\sec^2(20\pi x^5)} + \dfrac{2x}{\csc^2(20\pi x^5)}$?

A. 2 B. $200\pi x\left(\tan^2(20\pi x^5) - \cot^2(20\pi x^5)\right)$

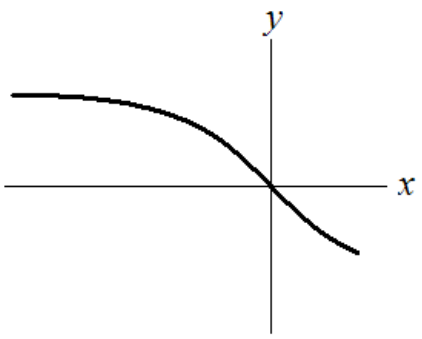
C. 0 D. $200\pi x$

8. Of the three graphs labeled I, II and III below, one is f, one is f' and one is f''. Which of the following correctly identifies the three graphs?

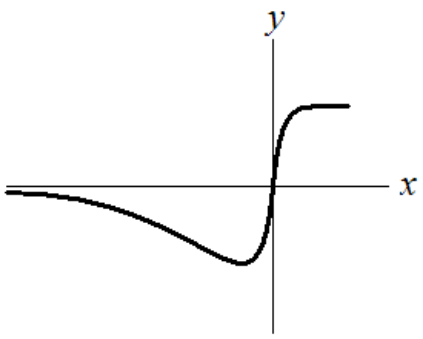
I.



II.



III.



A. $f = \text{II},\ f' = \text{III},\ f'' = \text{I}$
B. $f = \text{I},\ f' = \text{II},\ f'' = \text{III}$
C. $f = \text{II},\ f' = \text{I},\ f'' = \text{III}$
D. $f = \text{III},\ f' = \text{I},\ f'' = \text{II}$

9. What is the interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{100^n (x-4)^n}{(3n)!}$?

A. $\left(\frac{11}{3}, \frac{13}{3}\right)$ B. $(-5,13)$ C. $(-\infty,\infty)$ D. $\{4\}$

10. Integrate $\int \frac{14x^3}{x^2+1}\,dx$

A. $\frac{14}{3}\left(x^4-\tan^{-1}(x)\right)+C$ B. $7x^2-7+\tan^{-1}(x)+C$

C. $7x^2-\ln\left(x^2+1\right)+C$ D. $7\left(x^2-\ln\left(x^2+1\right)\right)+C$

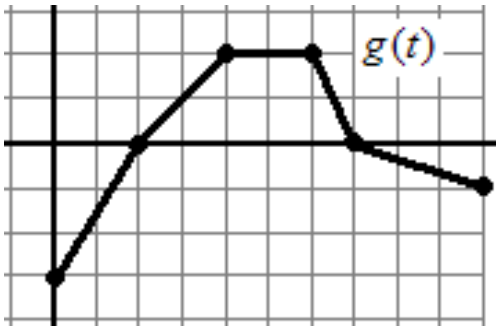
11. Evaluate $\int_{-\sqrt{3}}^{\sqrt{3}} \tan^{-1}(3x)\,dx$

A. 6π B. 0 C. $-\frac{2\pi}{3}$ D. $\frac{4\pi}{3}$

12. Evaluate the limit $\lim_{x\to 0+}\left(1+\frac{x}{3}\right)^{\frac{4}{5x}}$.

A. $\ln(0.2)$ B. $e^{4/15}$ C. $e^{1/3}$ D. ∞

13. The function shown below, $g(t)$, has a domain of $[0,10]$ and a range of $[-3,2]$.



The function $f(x)$ is defined as …

$$f(x)=\int_0^x g(t)\,dt\,.$$

What is the minimum value of $f(x)$ on the interval $[0,10]$?

A. –3 B. 0 C. 1 D. – 6

14. Solve the exact differential equation $\left(5y^2e^x\right)dx+\left(4+10ye^x\right)dy=0$.

A. $8y^2+5e^x=C$ B. $4ye^x+10y^2=C$
C. $4y^2+10ye^x=C$ D. $4y+5y^2e^x=C$

15. Evaluate the double integral $\int_0^{\sqrt{3\pi}}\int_0^x 5\sin(x^2)\,dy\,dx$.

A. 5 B. 10 C. 0 D. 6π

16. Determine the value of the area bounded between the graph of $y=\sin x$ and the graph of its tangent line at $x=0$ from the values $x=0$ to $x=\pi$.

A. $\frac{2}{3}\left(3\pi^2-1\right)$ B. $\frac{1}{2}\left(\pi^2-4\right)$ C. $\frac{1}{3}\left(\pi^2-2\right)$ D. $\frac{1}{4}\left(2\pi^2+1\right)$

17. Integrate $\int x\sec^2(7x)\,dx$

A. $\frac{1}{7}\sec 7x\tan 7x-\frac{1}{7}\ln\left|\sec 7x\right|+C$
B. $\frac{1}{7}x\tan 7x+\frac{1}{49}\ln\left|\cos 7x\right|+C$
C. $\frac{1}{7}\sec^2 7x+\frac{1}{7}\ln\left|\cos 7x\right|+C$
D. $\frac{1}{14}x^2\tan 7x+\frac{1}{7}\ln\left|\cos 7x\right|+C$

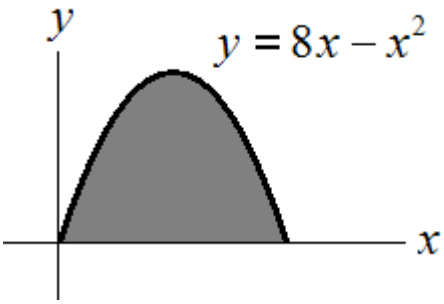
18. Given the vector function $\mathbf{r}(t)=(4t-3t^2)\,\mathbf{i}+(2\cos^{-1}t)\,\mathbf{j}$, evaluate the unit tangent vector $\mathbf{T}(0)$.

A. $-\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$ B. $\frac{2}{\sqrt{29}}\mathbf{i}+\frac{5}{\sqrt{29}}\mathbf{j}$ C. $\frac{2}{\sqrt{5}}\mathbf{i}-\frac{1}{\sqrt{5}}\mathbf{j}$ D. $-\frac{3}{\sqrt{10}}\mathbf{i}-\frac{1}{\sqrt{10}}\mathbf{j}$

19. For the first-order ODE $\sin(x)\dfrac{dy}{dx}+\cos(x)\ y=\sin(x)e^{\sec^2 x}$, determine the integrating factor $\mu(x)$.

A. $e^{-\sin x}$ B. $\left|\sin(x)\right|$ C. $e^{\tan x}$ D. $\left|\tan x\right|$

20. Determine the volume of revolving the region shown below about the line $x=12$.



A. $1024\pi/3$ cubic units

B. $4096\pi/3$ cubic units

C. $5132\pi/3$ cubic units

D. $6892\pi/3$ cubic units

21. A steady volume of sand is poured from a conveyor belt into a quarry below. The pile makes the shape of a right circular cone with a height equal to 2/3 of the base diameter. At the instant the cone is 8 meters high, the height is growing at a rate of 0.6 m/min. What is the flow rate of the sand leaving the conveyor belt?

A. 48.7 m^3/min B. 58.8 m^3/min C. 67.9 m^3/min D. 76.1 m^3/min

22. What is the area inside one petal of the three-leaf rose $r=12\cos(3\theta)$?

A. 12π B. 6π C. 8π D. 18π

23. What is the area of the parallelogram spanned by the vectors $\mathbf{v} = 4\mathbf{i} - 7\mathbf{j}$ and $\mathbf{w} = -3\mathbf{i}$?

A. $\sqrt{74}$ sq. units B. $\sqrt{53}$ sq.units C. 28 sq. units D. 21 sq.units

24. Integrate $\int \frac{8}{6t - t^2}\, dt$ is

A. $\frac{2}{3}\ln\left| t^2 - 6t \right| + C$ B. $\frac{4}{3}\ln\left| \frac{t}{t-6} \right| + C$

C. $\frac{4}{3}\ln\left| \frac{t-6}{t} \right| + C$ D. $\frac{4}{3}\left(t - \tan^{-1}(3-t)\right) + C$

25. Evaluate the improper integral $\int_{-1}^{1} \ln|2x|\, dx$.

A. $4 + \ln 2$ B. $2 - \ln 8$ C. $-2 + \ln 4$ D. Divergent