TMATYC - Calculus B Test – 2011

Instructions for the Answer Sheet

DO NOT BEGIN UNTIL YOU ARE TOLD TO DO SO

To the student:

Complete **all** information on answer sheet. Carefully answer the eligibility questions. You will be disqualified if you take an incorrect test. If you are unsure of your eligibility status, ask your test monitor **NOW**, before starting the test. No questions may be asked once the test begins.

You have one hour to take this test. You are allowed to use a non-symbolic calculator (such as the TI-83, TI-84, or TI-86). Calculators that perform symbolic manipulations are **not** allowed (these include the TI-89, TI-92, or TI-Nspire). Blank scratch paper is allowed. No books, notes, or any other electronic devices are allowed. Please refrain from using any cell phone during the test. Such devices should be muted or put on silent mode.

There are 25 questions on the test. Each question is worth 4 points for a correct answer, but 1 point will be subtracted for each incorrect answer. There is no penalty for unanswered questions.

You are not expected to answer every question in the time allowed. If you are having difficulty with a question, skip it and, if time permits, return to it after you finish the others.

Place the letter for your choice of the correct response on the answer sheet under the column entitled “Student’s Response”. **Write your letters in block capital form (i.e. write as A B C D E).**

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TMATYC - Calculus B Test – 2011
Answer Sheet

Name: _____ School: _____

Address: _____

Current Math Class: _____ Math Teacher: _____

	Student's Response	Scorer
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Email: _____

Phone: _____

Have you received a two-year or higher college degree? Yes No

For Scorer:

Number Correct = _____

Number Incorrect = _____

Number Blank = _____

Num Correct × 4 = _____

– Num Incorrect = _____

_____

Score on Test = _____

TMATYC
Calculus B Test
2011

1. Evaluate $\lim\limits_{t\to A+} 15000e^{\frac{-3}{A-t}}$

   A. 0 B. ∞ C. $-\infty$ D. 15000

2. Let $g\left(x\right) = \begin{cases} x^3 + k - 2 \text{ if } x \le 2 \\ kx^2 \quad \text{if } x > 2 \end{cases}$

   Determine the value of k such that $g(x)$ is continuous for all x.

   A. $k = 0$ B. $k = 1$ C. $k = 2$ D. $k = 3$

3. If $f\left(x\right) = \ln\left(\sin \pi x\right)$ then $f'\left(x\right) =$

   A. $\pi \cot \pi x$ B. $\dfrac{1}{\sin \pi x}$ C. $\dfrac{\pi \cos \pi x}{x}$ D. $\dfrac{\pi}{\cos \pi x}$

4. If $f\left(x\right) = \sqrt{9 - x}$ and $g\left(x\right) = x^3 + 1$ then $\left(f \circ g\right)'\left(x\right) =$

   A. $\dfrac{3\sqrt{9-x}}{2}$ B. $\dfrac{-3x^2}{2\sqrt{8-x^3}}$ C. $\dfrac{-1}{2\sqrt{9-x}} + 3x^2$ D. $\dfrac{-2}{3\sqrt[3]{9-x}}$

5. The position (in cm) of an object at time t (in seconds) is given by $s\left(t\right) = Ate^{-kt^2}$, where k and A are positive constants. The object moves backwards when its velocity is negative. When is the object moving backwards?

   A. $\left(-\infty, 0\right)$ B. $\left(\dfrac{-1}{\sqrt{2k}}, \dfrac{1}{\sqrt{2k}}\right)$ C. $\left(-\infty, \dfrac{-1}{\sqrt{2k}}\right) \cup \left(\dfrac{1}{\sqrt{2k}}, \infty\right)$ D. $\left(A, \infty\right)$

6. A bucket sits beneath a faucet from which water is pouring at a constant rate. Someone then begins to turn the faucet off. If $V(t)$ gives the volume of water in the bucket at time t, which of the following statements correctly describes the signs of $V'(t)$ and $V''(t)$?

A. $V'(t)$ is positive and $V''(t)$ is positive.
B. $V'(t)$ is positive and $V''(t)$ is negative.
C. $V'(t)$ is negative and $V''(t)$ is positive.
D. $V'(t)$ is negative and $V''(t)$ is negative.

7. Find all the values of x for which $g(x) = \dfrac{1}{x^2} + \ln x$ has a point of inflection.

A. $\sqrt{6}$ B. $0, \pm\sqrt{2}$ C. 1, 2 D. $\pm\sqrt{6}$

8. If $f(x) = x^2 \cos x$ then $f''(\pi) =$

A. 2 B. π^2 C. $\pi^2 - 2$ D. -2π

9. Find $f(x)$ subject to the following conditions: $f''(x) = 6x - 4$, $f(1) = 1$, $f'(0) = 3$

A. $f(x) = x^3 - 2x^2 + 3x + 1$
B. $f(x) = 6x^3 - 4x^2 - 3x + 1$
C. $f(x) = 2x^3 - 2x^2 + 3x - 2$
D. $f(x) = x^3 - 2x^2 + 3x - 1$

10. If $f(x) = x^2 \int_0^x \sin(t^2)\,dt$ then $f'(x) =$

A. $2x \int_0^x \sin(t^2)\,dt$
B. $x^2 \sin(x^2) + 2x \int_0^x \sin(t^2)\,dt$
C. $2x \sin(x^2)$
D. $x^2 \sin(t^2) + 2x \int_0^x 2t \cos(t^2)\,dt$

11. $\int e^3 dx =$

A. $e^3 x + C$ B. $e^3 + C$ C. $\dfrac{e^4}{4} + C$ D. $3e^2 + C$

12. If $\int_1^2 g(x)\,dx = 5$ and $\int_5^2 g(x) = 2$ then $\int_1^5 \big(3g(x)+1\big)\,dx =$

A. 10 B. 12 C. 13 D. 16

13. The average value of $f(x)$ over the interval $[a,b]$ is $\dfrac{1}{b-a}\int_a^b f(x)\,dx$. Find the value of p so that the average value of $f(x) = 2x + p$ over the interval $[0,p]$ is 10.

A. $\dfrac{9}{2}$ B. 5 C. $\dfrac{20}{3}$ D. 9

14. $\int \cos^3 t \sin^6 t\,dt =$

A. $\dfrac{1}{28}\cos^4 t \sin^7 t + C$ B. $\dfrac{1}{7}\sin^7 t - \dfrac{1}{9}\sin^9 t + C$

C. $3\cos^2 t \sin^5 t\big(2\cos^2 t - \sin^2 t\big) + C$ D. $\dfrac{1}{4}\cos^4 t - \dfrac{1}{8}\cos^8 t + C$

15. $\int x\cos 2x\,dx =$

A. $\dfrac{x^2 \sin 2x}{4} + C$ B. $\cos 2x - 2x\sin 2x + C$

C. $2x\sin x + 2\cos x + C$ D. $\dfrac{1}{2}x\sin 2x + \dfrac{1}{4}\cos 2x + C$

16. What is the volume of the solid created by revolving the region bounded between the curves $y = 2x - x^2$ and $y = 0$ about the line $x = 5$?

A. $\dfrac{16\pi}{3}$ B. 2π C. $\dfrac{32\pi}{3}$ D. $\dfrac{64\pi}{3}$

17. For what values of k does $y = \dfrac{3k}{2+5t}$ satisfy the differential equation $y'' + yy' = 0$?

A. 0 & 10/3 B. 0 & 3/2 C. –1 & 3/2 D. 0 & 50/3

18. Use trigonometric substitution to evaluate $\displaystyle\int \frac{dx}{\left(x^2+4\right)^{3/2}}$

A. $\dfrac{x}{4\sqrt{x^2+4}}+C$

B. $\dfrac{2}{5\left(x^2+4\right)^{5/2}}+C$

C. $3\ln\left|x+2\right|-\dfrac{x}{\sqrt{x^2+4}}+C$

D. $\ln\left|\dfrac{x}{\sqrt{x^2+4}}+\dfrac{4}{x}\right|+C$

19. Use partial fraction decomposition to evaluate $\displaystyle\int \frac{11x-3}{x^2-x-6}dx$

A. $2\ln\left|x-6\right|+9\ln\left|x+1\right|+C$

B. $\ln\left|x-6\right|+10\ln\left|x+1\right|+C$

C. $6\ln\left|x-3\right|+5\ln\left|x+2\right|+C$

D. $4\ln\left|x-3\right|-7\ln\left|x+2\right|+C$

20. Given the recursively-defined sequence $a_1=2, a_2=3, a_n=a_{n-1}-a_{n-2}$ find a_{2011}.

A. –3 B. –2 C. 1 D. 2

21. What is the sum of the series $\dfrac{\pi}{3}-\dfrac{\pi^3}{3^3\cdot 3!}+\dfrac{\pi^5}{3^5\cdot 5!}-\dfrac{\pi^7}{3^7\cdot 7!}+\dfrac{\pi^9}{3^9\cdot 9!}-\cdots$?

A. $\dfrac{\sqrt{3}}{2}$ B. $\sqrt{\pi}$ C. $\dfrac{1}{2}$ D. $e^{\pi/3}$

22. Find the first four nonzero terms in a power series expansion about $x=0$ for the solution to the initial value problem $y''-2xy'+4y=0,\quad y(0)=2,\quad y'(0)=3$.

A. $y=2+3x-4x^2-3x^3$

B. $y=2+3x-3x^2-x^3$

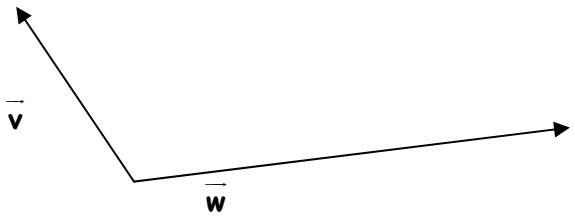
C. $y=2+3x-4x^2-x^3$

D. $y=2+3x-x^2-4x^3$

23. $\displaystyle\int_0^1\int_0^y\left(8xy+e^x\right)dx\,dy=$

A. $e-1$ B. e C. $e+1$ D. $e+15$

24. $\vec{v}$ and $\vec{w}$ are the vectors drawn below. The magnitude of $\vec{w}$ is 5. Which of the following is a true statement about the vector projection of $\vec{v}$ onto $\vec{w}$?



The vector projection of $\vec{v}$ onto $\vec{w}$ is a

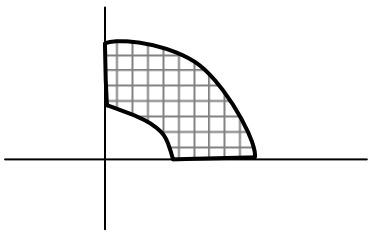
A. positive multiple of $\vec{w}$ and its magnitude exceeds 5.

B. positive multiple of $\vec{w}$ and its magnitude is less than 5.

C. negative multiple of $\vec{w}$ and its magnitude exceeds 5.

D. negative multiple of $\vec{w}$ and its magnitude is less than 5.

25. Let the curve C be the boundary of the region between the quarter circles of radii a and b, oriented in the positive direction. C is drawn below:



Evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where the vector field $\vec{F}$ is defined by

$\vec{F} = \left(4 + 3e^{5\cos^2(x)}\right)\vec{i} + \left(\sin(e^y) + 3x^2\right)\vec{j}$.

A. 0 B. $2\left(b^3 - a^3\right)$ C. $\dfrac{\pi\left(b^2 - a^2\right)}{4}$

D. It is not possible to give an exact answer because the integrals of $e^{5\cos^2(x)}$ and $\sin(e^y)$ cannot be computed exactly.