

TMATYC - Calculus B Test – 2012

Instructions for the Answer Sheet

DO NOT BEGIN UNTIL YOU ARE TOLD TO DO SO

To the student:

Complete **all** information on answer sheet. Carefully answer the eligibility questions. You will be disqualified if you take an incorrect test. If you are unsure of your eligibility status, ask your test monitor **NOW**, before starting the test. No questions may be asked once the test begins.

You have one hour to take this test. You are allowed to use a non-symbolic calculator (such as the TI-83, TI-84, or TI-86). Calculators that perform symbolic manipulations are **not** allowed (these include the TI-89, TI-92, or TI-Nspire). Blank scratch paper is allowed. No books, notes, or any other electronic devices are allowed. Please refrain from using any cell phone during the test. Such devices should be muted or put on silent mode.

There are 25 questions on the test. Each question is worth 4 points for a correct answer, but 1 point will be subtracted for each incorrect answer. There is no penalty for unanswered questions.

You are not expected to answer every question in the time allowed. If you are having difficulty with a question, skip it and, if time permits, return to it after you finish the others.

Place the letter for your choice of the correct response on the answer sheet under the column entitled "Student's Response". **Write your letters in block capital form (i.e. write as A, B, C, or D).**

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Answer Sheet

Name: _____ School: _____

Address: _____

Current Math Class: _____

Math Teacher: _____

	Student's Response	Scorer
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Email: _____

Phone: _____

Have you received a two-year or higher college degree? Yes No

For Scorer:

Number Correct = _____

Number Incorrect = _____

Number Blank = _____

Num Correct \times 4 = _____

– Num Incorrect = _____

Score on Test = _____

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1. Find the derivative of $y = x^{\sin x}$.

A. $\frac{dy}{dx} = x^{\sin x} \cos x$

C. $\frac{dy}{dx} = \left(\ln x \cos x + \frac{\sin x}{x} \right) x^{\sin x}$

B. $\frac{dy}{dx} = x^{\sin x - 1} \sin x$

D. $\frac{dy}{dx} = \frac{\sin x}{x} + \ln(\sin x) + x \cos x$

2. Evaluate the integral $\int_1^{e^{\pi/2}} \sin(\ln x) dx$

A. 1

B. $\frac{e^{\pi/2} + 1}{2}$

C. $\frac{\pi}{2} e^{\pi/2}$

D. $1 - \ln \frac{\pi}{2}$

3. Suppose $\int_0^8 f(x) dx = 12$. What is the value of $\int_0^2 f(x^3) x^2 dx$?

A. 4

B. 12

C. 3

D. 36

4. The government decides to introduce a new design for the ten dollar bills (\$10). We can model the proportion, $P(t)$ of new bills in circulation as a function of the number of days passed with the following equation.

$$P'(t) = .005(1 - P(t))$$

The initial proportion of new bills in circulation is zero (0). Approximately how many days will it take for the proportion of new bills to reach 80%?

A. 110 days

B. 320 days

C. 540 days

D. 900 days

5. Let

$$H(n) = \int_0^{\infty} x^n e^{-x/2} dx$$

and find $H(2)$.

A. 4

B. 6

C. 8

D. 16

6. Evaluate the following limit: $\lim_{x \rightarrow 0} \frac{x - xe^x}{\cos x - 1}$

A. 0

B. 1

C. 2

D. limit does not exist

7. For what value of the constant k will $g(x) = x \sin(kx)$ best approximate $f(x) = 1 - \cos x$ for x near 0.

- A. $k = 1$ B. $k = \frac{1}{2}$ C. $k = 2$ D. $k = \frac{1}{3}$

8. Let $x(t) = te^t$ and $y(t) = t - t^2$. Find the slope of the tangent line to the curve at the point $(e, 0)$.

- A. $2e$ B. $-2e$ C. $-\frac{1}{2e}$ D. $\frac{1}{2}$

9. Let R be the region bounded by $x(t) = t^2$, $y(t) = t^3 - 4t$ for $-2 \leq t \leq 2$. Evaluate the integral.

$$\iint_R x \, dx \, dy$$

- A. 0 B. $\frac{1024}{35}$ C. $\frac{16}{3}$ D. $\frac{1920}{343}$

10. Let R be the region inside the circle $x^2 + y^2 = 9$ and outside the circle $x^2 + y^2 = 1$. Evaluate the integral.

$$\iint_R e^{x^2+y^2} \, dx \, dy$$

- A. $\pi(e^9 - e)$ B. $\pi(e^3 - e)$ C. $\pi\left(\frac{e^9}{3} - e\right)$ D. $\pi(3e^3 - e)$

11. Evaluate the integral.

$$\int \frac{dx}{x^2\sqrt{6+x^2}}$$

- A. $\frac{x\sqrt{6+x^2}}{6} + C$ B. $\frac{6x}{\sqrt{6+x^2}} + C$ C. $\frac{(6+x^2)^{3/2}}{6x} + C$ D. $-\frac{\sqrt{6+x^2}}{6x} + C$

12. Find the area inside the six leaves of the polar curve $r^2 = 2 \sin(3\theta)$.

- A. 4 B. $\frac{2}{3}$ C. 12 D. $\frac{4}{9}$

13. Evaluate the integral.

$$\int_1^4 \frac{1}{(x-2)^2} dx$$

- A. $-\frac{3}{2}$ B. $\frac{3}{2}$ C. $\frac{1}{2}$ D. does not exist

14. Find the volume of the solid obtained by revolving the region in the first quadrant bounded by $x = y^2$, $y = 2$, and $x = 0$ about the line $y = -2$.

- A. $\frac{56}{3}\pi$ B. $\frac{8}{3}\pi$ C. 56π D. 8π

15. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P ?

- A. 4π Km/min B. $\frac{80\pi}{3}$ Km/min C. $\frac{27\pi}{4}$ Km/min D. 120π km/min

16. Solve the boundary value problem :

$$y'' = 3x - 2; \quad y(0) = 2; \quad y'(1) = -3$$

- A. $y(x) = \frac{x^3}{2} - x^2 + 2$ C. $y(x) = \frac{x^3}{2} - x^2 - \frac{5x}{2}$
B. $y(x) = \frac{x^3}{2} - x^2 - \frac{5x}{2} + 2$ D. $y(x) = \frac{x^3}{2} - x^2 + \frac{5x}{2} + 2$

17. Find the Equation for the plane passing through the points $P_1(3,1,-2)$; $P_2(-1,2,4)$; $P_3(2,-1,1)$

- A. $-5x - 2y + 3z = 11$ B. $5x + 2y - 3z = -11$ C. $-5x + 2y - 3z = 11$ D. $5x + 2y + 3z = 11$

18. Evaluate $\int_0^{\pi/2} |\cos x - \sin x| dx$

- A. $2\sqrt{2}$ B. $2(\sqrt{2} - 1)$ C. $\frac{\sqrt{3}}{2}(\sqrt{2} - 1)$ D. $\frac{\sqrt{3}}{2}$

19. Sum the geometric Series $\sum_{n=0}^{\infty} \frac{3^{2n+1}}{2(11^{n+1})}$

- A. $\frac{3}{2}$ B. $\frac{9}{11}$ C. $\frac{3}{22}$ D. $\frac{3}{4}$

20. Determine the Area of the region bounded by the curve $y = 6x - x^2 - 4$ and the line $y = x$

- A. 12 B. $\frac{15}{2}$ C. 6 D. $\frac{9}{2}$

21. Evaluate $\int_{2^5}^{3^5} \frac{dx}{x - x^{3/5}}$

- A. $\frac{1}{2} \ln\left(\frac{4}{3}\right)$ B. $-\frac{1}{2}$ C. $\frac{1}{2} \ln\left|5 - \frac{8}{3}\right|$ D. $\frac{5}{2} \ln\left(\frac{8}{3}\right)$

22. Find $\int \frac{dx}{x^7 - x}$ (Hint: do not use partial fraction decomposition)

- A. $\frac{1}{7} \ln \left| \frac{x^6 - 1}{x^6} \right| + c$ B. $\frac{3}{7} \ln |x^7 - 1|$
C. $\ln |x^6 - 1| + c$ D. $\frac{1}{6} \ln \left| \frac{x^6 - 1}{x^6} \right| + c$

23. Evaluate $\int_1^4 \frac{dx}{\sqrt{x} + x}$

- A. $\frac{1}{2}$ B. $\frac{1}{2} \ln \frac{3}{4}$ C. $2 \ln \frac{3}{2}$ D. $\frac{3}{4}$

24. Use partial fraction decomposition to evaluate: $\int \frac{7x - 14}{x^2 - 3x - 10}$

- A. $\ln|x^2 - 3x - 10| + C$ B. $\frac{\frac{7}{2}x^2 - 14x}{\frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x} + C$
C. $\frac{7}{2x - 3} + C$ D. $4 \ln|x + 2| + 3 \ln|x - 5| + C$

25. Find $\lim_{x \rightarrow 1} \frac{\frac{1}{x} - 1}{\sqrt{x} - 1}$

- A. 0 B. $-\infty$ C. 1 D. -2