

TMATYC - Calculus A Test - 2013

1. If the tangent line to the graph of the function $f(x)$ at the point $(1, -8)$ passes through the point $(-1, -4)$, then $f'(1) =$

- A. -8 B. -2 C. 0 D. $\frac{1}{2}$ E. 1

2. Find the following limit, if it exists: $\lim_{x \rightarrow -3} \frac{2x^2 + 7x + 3}{x + 3}$

- A. -5 B. 0 C. 2 D. 7 E. Does not exist

3. If $a, b, c,$ and d are all positive real numbers, then $\lim_{x \rightarrow \infty} \frac{a - \frac{20b}{3x}}{e^{-cx} + d} =$

- A. 0 B. $\frac{20b-3a}{3d}$ C. $\frac{a}{d}$ D. ∞ E. $-\infty$

4. Find all the values of k that make the function $g(x)$ continuous.

$$g(x) = \begin{cases} 2kx + 3 & \text{if } x \leq k \\ 4x + 9 & \text{if } x > k \end{cases}$$

- A. $k = -1, 2$ B. $k = 1, 3$ C. $k = -1, 3$ D. $k = 0$ E. $k = 2$

5. Which of the following is an equation of the tangent line to $y = \sin x + \cos x$ at $x = \pi$?

- A. $y = -x + \pi + 1$ B. $y = x - \pi + 1$ C. $y = -x - \pi + 1$ D. $y = x - \pi - 1$ E. $y = -x + \pi - 1$

6. Find y'' if $y = x \sin x$

- A. $-\sin x$ B. $1 - x \cos x$ C. $\sin x - x \cos x$ D. $2 \cos x - x \sin x$ E. $x \cos x + \sin x$

7. A body is moving in simple harmonic motion with position function $s(t) = 3 + \sin\left(\frac{3t}{4}\right)$. At which of the following times is the acceleration of the body zero?

- A. $t = \frac{\pi}{4}$ B. $t = \frac{\pi}{2}$ C. $t = \frac{3\pi}{4}$ D. $t = \frac{4\pi}{3}$ E. $t = \pi$

8. Let f and g be functions that satisfy $f'(2) = 6$, $g'(2) = 3$, $f(2) = 7$, and $g(2) = 1$. Find $h'(2)$ if $h(x) = \sqrt{f(x) + 2g(x)}$

- A. 2 B. 3 C. $2\sqrt{3}$ D. 6 E. $4\sqrt{3}$

9. Find $\frac{dy}{dx}$ if $y = \cos^2(x^3 + x^2)$

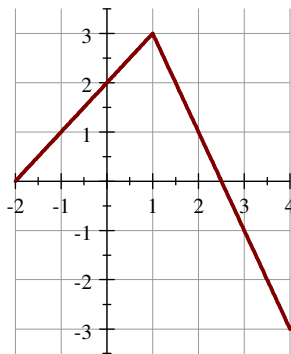
- A. $-2 \sin(3x^2 + 2x)$ B. $-\sin^2(3x^2 + 2x)$ C. $-2(3x^2 + 2x) \cos(x^3 + x^2) \sin(x^3 + x^2)$
 D. $(3x^2 + 2x) \sin^2(x^3 + x^2)$ E. $\sin^2(x^3 + x^2) + \cos^2(3x^2 + 2x)$

10. If $y = \frac{2 - ax}{bx - 3}$, then $\frac{dy}{dx} =$

- A. $-\frac{a}{b}$ B. $\frac{3a - 2b}{(bx - 3)^2}$ C. $\frac{2ab(3 - x)}{(bx - 3)^2}$ D. $\frac{(2b - 3a)(x - 1)}{(bx - 3)^2}$ E. $\frac{2 - a}{(b - 3)^2}$

11. If $a < 0$, then the graph of $y = ax^3 + 3x^2 + 4x + 5$ is concave up on the interval
 A. $(-\infty, -\frac{1}{a})$ B. $(-\infty, \frac{1}{a})$ C. $(\frac{1}{a}, \infty)$ D. $(-\frac{1}{a}, \infty)$ E. $(-\infty, \infty)$
12. How many points of inflection does the graph of $y = x^5 - 5x^4 + 3x + 7$ have?
 A. 0 B. 1 C. 2 D. 3 E. 4
13. If the linearization of $y = \sqrt[3]{x}$ at $x = 64$ is used to approximate $\sqrt[3]{66}$, the percentage error is
 A. 0.04% B. 0.01% C. 1% D. 0.4% E. 1.26%
14. A particle is moving around the unit circle (the circle of radius 1 centered at the origin). At the point $(0.6, 0.8)$ the particle has horizontal velocity $\frac{dx}{dt} = 5$. What is the vertical velocity?
 A. -3.875 B. -2.25 C. -3.75 D. 6.67 E. 5.2
15. An observer 70 meters south of a railroad crossing watches an eastbound train travelling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection.
 A. 57.6 B. 57.9 C. 58.3 D. 59.2 E. 60.0
16. Let $f(x) = \frac{1}{1+x}$. What is the n th derivative of f ?
 A. $n!(1+x)^{n+1}$ B. $(-1)^n n!(1+x)^{n+1}$ C. $-\frac{n!}{(1+x)^{n+1}}$
 D. $\frac{n!}{(1+x)^{n+1}}$ E. $\frac{(-1)^n n!}{(1+x)^{n+1}}$
17. A rectangle is inscribed in a semicircle of radius 10 centimeters with one side lying on the diameter of the semicircle. What is the maximum possible area (in square centimeters) of the rectangle?
 A. $60\sqrt{5}$ B. $5\sqrt{5}$ C. $5\sqrt{2}$ D. 145 E. 100
18. At what point does the line normal to the curve $x^2y^3 + y + 2 = 0$ at $(1, -1)$ intersect the line $2x - 3y + 7 = 0$?
 A. $(-\frac{1}{2}, 2)$ B. $(-\frac{2}{3}, \frac{17}{9})$ C. $(-5, -1)$ D. $(\frac{1}{3}, \frac{23}{9})$ E. $(-23, -13)$
19. If the line $y = -x + 8$ is tangent to the graph of $g(x) = ax^2 + 7x$, then what is the value of a ?
 A. 2 B. $\frac{1}{2}$ C. $-\frac{7}{8}$ D. -2 E. Cannot be determined
20. Let $u(x) = \sqrt{x^2 + 9}$ and $v(x) = 3x^3 - 2x$. What is the value of $\frac{du}{dv}$ at $x = 4$?
 A. $\frac{1}{1420}$ B. $\frac{3}{755}$ C. $\frac{4}{855}$ D. $\frac{2}{355}$ E. $\frac{4}{25}$

Use the graph g shown below to answer Question #21



Graph of g for #21

21. Using the graph of g above, find $h'(2)$ if $h(x) = x^3 g(x)$.
- A. 8 B. -2 C. -4 D. 12 E. Not enough information to find $h'(2)$
22. If $f(x) = \int_{\pi/2}^x (\sqrt[3]{\sin t}) dt$ then at what value of x in the interval $[0, 2\pi]$ is $f(x)$ a maximum?
- A. 0 B. $\frac{\pi}{2}$ C. 2π D. π E. $\frac{\pi}{3}$
23. A particle moves on the x -axis with velocity $v(t) = \sin 2t$ at time t . At $t = 0$, the x -coordinate of the position of the particle is 2. What is the x -coordinate of the position at $t = \frac{\pi}{2}$?
- A. $\cos 4$ B. 3 C. $\cos 2$ D. 2 E. 0
24. If the derivative of the function $f(x)$ exists at $x = a \neq 0$ then $\frac{f(a)f'(a)}{a}$ is always equal to
- A. $\lim_{x \rightarrow a} \frac{f^2(x) - f^2(a)}{x^2 - a^2}$ B. $\lim_{x \rightarrow a} \frac{[f(x) - f(a)]^2}{x - a}$ C. $\lim_{x \rightarrow a} \frac{f^2(x) - f^2(a)}{x - a}$
- D. $\lim_{x \rightarrow a} \left[\frac{f(x) - f(a)}{x - a} \right]^2$ E. $\lim_{x \rightarrow a} \frac{f^2(x)}{x}$
25. If y is a function of x such that $xy^2 + x = (y - yx^2)y'$ for $-\frac{1}{2} < x < \frac{1}{2}$ and $y(0) = 1$, then $y =$
- A. $\sqrt{\frac{1+x^2}{1-2x^2}}$ B. $\sqrt{\frac{1+x^2}{1-x^2}}$ C. $\sqrt{1+x^2}$ D. $1 + x\sqrt{1+x^2}$ E. $(1+x^2)\sqrt{1-x^2}$