TMATYC - Calculus A Test - 2013

1. If the tangent line to the graph of the function $f(x)$ at the point $(1,-8)$ passes through the point $(-1,-4)$, then $f'(1) =$

A. -8 B. -2 C. 0 D. $\frac{1}{2}$ E. 1

2. Find the following limit, if it exists: $\lim\limits_{x\to -3} \dfrac{2x^2+7x+3}{x+3}$

A. -5 B. 0 C. 2 D. 7 E. Does not exist

3. If a, b, c, and d are all positive real numbers, then $\lim\limits_{x\to\infty} \dfrac{a-\frac{20b}{3x}}{e^{-cx}+d} =$

A. 0 B. $\frac{20b-3a}{3d}$ C. $\frac{a}{d}$ D. ∞ E. $-\infty$

4. Find all the values of k that make the function $g(x)$ continuous.

$$g(x) = \begin{cases} 2kx+3 & \text{if } x \le k \\ 4x+9 & \text{if } x > k \end{cases}$$

A. $k=-1,2$ B. $k=1,3$ C. $k=-1,3$ D. $k=0$ E. $k=2$

5. Which of the following is an equation of the tangent line to $y=\sin x+\cos x$ at $x=\pi$?

A. $y=-x+\pi+1$ B. $y=x-\pi+1$ C. $y=-x-\pi+1$ D. $y=x-\pi-1$ E. $y=-x+\pi-1$

6. Find y'' if $y=x\sin x$

A. $-\sin x$ B. $1-x\cos x$ C. $\sin x-x\cos x$ D. $2\cos x-x\sin x$ E. $x\cos x+\sin x$

7. A body is moving in simple harmonic motion with position function $s(t)=3+\sin\left(\frac{3t}{4}\right)$. At which of the following times is the acceleration of the body zero?

A. $t=\frac{\pi}{4}$ B. $t=\frac{\pi}{2}$ C. $t=\frac{3\pi}{4}$ D. $t=\frac{4\pi}{3}$ E. $t=\pi$

8. Let f and g be functions that satisfy $f'(2)=6$, $g'(2)=3$, $f(2)=7$, and $g(2)=1$. Find $h'(2)$ if $h(x)=\sqrt{f(x)+2g(x)}$

A. 2 B. 3 C. $2\sqrt{3}$ D. 6 E. $4\sqrt{3}$

9. Find $\dfrac{dy}{dx}$ if $y=\cos^2(x^3+x^2)$

A. $-2\sin(3x^2+2x)$ B. $-\sin^2(3x^2+2x)$ C. $-2(3x^2+2x)\cos(x^3+x^2)\sin(x^3+x^2)$

D. $(3x^2+2x)\sin^2(x^3+x^2)$ E. $\sin^2(x^3+x^2)+\cos^2(3x^2+2x)$

10. If $y=\dfrac{2-ax}{bx-3}$, then $\dfrac{dy}{dx} =$

A. $-\dfrac{a}{b}$ B. $\dfrac{3a-2b}{(bx-3)^2}$ C. $\dfrac{2ab(3-x)}{(bx-3)^2}$ D. $\dfrac{(2b-3a)(x-1)}{(bx-3)^2}$ E. $\dfrac{2-a}{(b-3)^2}$

11. If $a < 0$, then the graph of $y = ax^3 + 3x^2 + 4x + 5$ is concave up on the interval

A. $(-\infty, -\frac{1}{a})$ B. $(-\infty, \frac{1}{a})$ C. $(\frac{1}{a}, \infty)$ D. $(-\frac{1}{a}, \infty)$ E. $(-\infty, \infty)$

12. How many points of inflection does the graph of $y = x^5 - 5x^4 + 3x + 7$ have?

A. 0 B. 1 C. 2 D. 3 E. 4

13. If the linearization of $y = \sqrt[3]{x}$ at $x = 64$ is used to approximate $\sqrt[3]{66}$, the percentage error is

A. 0.04% B. 0.01% C. 1% D. 0.4% E. 1.26%

14. A particle is moving around the unit circle (the circle of radius 1 centered at the origin). At the point $(0.6, 0.8)$ the particle has horizontal velocity $\frac{dx}{dt} = 5$. What is the vertical velocity?

A. –3.875 B. –2.25 C. –3.75 D. 6.67 E. 5.2

15. An observer 70 meters south of a railroad crossing watches an eastbound train travelling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection.

A. 57.6 B. 57.9 C. 58.3 D. 59.2 E. 60.0

16. Let $f(x) = \dfrac{1}{1+x}$. What is the nth derivative of f?

A. $n!(1+x)^{n+1}$ B. $(-1)^n n!(1+x)^{n+1}$ C. $-\dfrac{n!}{(1+x)^{n+1}}$

D. $\dfrac{n!}{(1+x)^{n+1}}$ E. $\dfrac{(-1)^n n!}{(1+x)^{n+1}}$

17. A rectangle is inscribed in a semicircle of radius 10 centimeters with one side lying on the diameter of the semicircle. What is the maximum possible area (in square centimeters) of the rectangle?

A. $60\sqrt{5}$ B. $5\sqrt{5}$ C. $5\sqrt{2}$ D. 145 E. 100

18. At what point does the line normal to the curve $x^2y^3 + y + 2 = 0$ at $(1, -1)$ intersect the line $2x - 3y + 7 = 0$?

A. $\left(-\frac{1}{2}, 2\right)$ B. $\left(-\frac{2}{3}, \frac{17}{9}\right)$ C. $(-5, -1)$ D. $\left(\frac{1}{3}, \frac{23}{9}\right)$ E. $(-23, -13)$

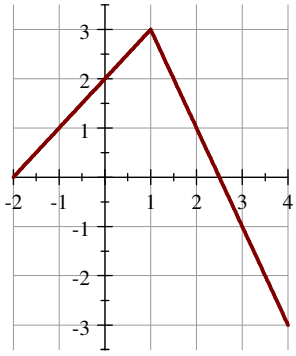
19. If the line $y = -x + 8$ is tangent to the graph of $g(x) = ax^2 + 7x$, then what is the value of a?

A. 2 B. $\frac{1}{2}$ C. $-\frac{7}{8}$ D. –2 E. Cannot be determined

20. Let $u(x) = \sqrt{x^2 + 9}$ and $v(x) = 3x^3 - 2x$. What is the value of $\dfrac{du}{dv}$ at $x = 4$?

A. $\frac{1}{1420}$ B. $\frac{3}{755}$ C. $\frac{4}{855}$ D. $\frac{2}{355}$ E. $\frac{4}{25}$

Use the graph g shown below to answer Question #21



Graph of g for #21

21. Using the graph of g above, find $h'(2)$ if $h(x) = x^3 g(x)$.

A. 8 B. –2 C. –4 D. 12 E. Not enough information to find $h'(2)$

22. If $f(x) = \int_{\pi/2}^{x} \left(\sqrt[3]{\sin t}\right) dt$ then at what value of x in the interval $[0, 2\pi]$ is $f(x)$ a maximum?

A. 0 B. $\frac{\pi}{2}$ C. 2π D. π E. $\frac{\pi}{3}$

23. A particle moves on the x-axis with velocity $v(t) = \sin 2t$ at time t. At $t = 0$, the x-coordinate of the position of the particle is 2. What is the x-coordinate of the position at $t = \frac{\pi}{2}$?

A. $\cos 4$ B. 3 C. $\cos 2$ D. 2 E. 0

24. If the derivative of the function $f(x)$ exists at $x = a \neq 0$ then $\dfrac{f(a)f'(a)}{a}$ is always equal to

A. $\lim\limits_{x \to a} \dfrac{f^2(x) - f^2(a)}{x^2 - a^2}$ B. $\lim\limits_{x \to a} \dfrac{[f(x) - f(a)]^2}{x - a}$ C. $\lim\limits_{x \to a} \dfrac{f^2(x) - f^2(a)}{x - a}$

D. $\lim\limits_{x \to a} \left[\dfrac{f(x) - f(a)}{x - a} \right]^2$ E. $\lim\limits_{x \to a} \dfrac{f^2(x)}{x}$

25. If y is a function of x such that $xy^2 + x = (y - yx^2)y'$ for $-\frac{1}{2} < x < \frac{1}{2}$ and $y(0) = 1$, then $y =$

A. $\sqrt{\dfrac{1 + x^2}{1 - 2x^2}}$ B. $\sqrt{\dfrac{1 + x^2}{1 - x^2}}$ C. $\sqrt{1 + x^2}$ D. $1 + x\sqrt{1 + x^2}$ E. $(1 + x^2)\sqrt{1 - x^2}$