TMATYC - Calculus A Test - 2014

1. For the function

$$f(x) = \begin{cases} x^2 - 5 & \text{if} \quad x < -2 \\ x + 3 & \text{if} \quad -2 \le x \le 4 \\ -x^2 + 20 & \text{if} \quad x > 4 \end{cases}$$

find $\lim_{x \to -2} f(x)$

A. -4 B. -1 C. 1 D. 16 E. Does not exist

2. On what interval(s) is $f(x) = \dfrac{x-7}{x^2-49}$ continuous?

A. $(-7,7)$ B. $(-\infty,-7) \cup (-7,7) \cup (7,\infty)$ C. $(-7,7) \cup (7,\infty)$
D. $(-\infty,7) \cup (7,\infty)$ E. $(-\infty,-7) \cup (-7,\infty)$

3. Find the average value of $f(x) = x^3 + 2x - 1$ over the interval $[2,4]$.

A. 36 B. 30 C. 35 D. 32 E. 41

4. Find $\dfrac{ds}{dt}$ for $s = \dfrac{t^2+2}{3-t^2}$

A. $\dfrac{2t}{(3-t^2)^2}$ B. $\dfrac{10t}{(3-t^2)^2}$ C. $\dfrac{-4t^3+2t}{(3-t)^2}$ D. -1 E. 0

5. Find the equation of the line tangent to the graph of $y = 3e^x - 1$ at the point $(0,2)$.

A. $y = 3x$ B. $y = 3e^x + 2$ C. $y = 3x - 6$ D. $y = 3x + 2$ E. $y = 2x + 2$

6. Find the vertical and horizontal asymptotes of the function

$$f(x) = \frac{x-3}{x^2-2x-8}$$

A. Vertical: $x = -2, x = 4$ Horizontal: $y = 0$ B. Vertical: $x = 0$ Horizontal: $y = -2, y = 4$
C. Vertical: $x = 3, x = -2, x = 4$ Horizontal: $y = 0$ D. Vertical: $x = -4, x = 2$ Horizontal: none
E. Vertical: $x = -2, x = 4$ Horizontal: none

7. Find y' if $y = (x^2+3)^4(2x^3-5)^3$

A. $12(x^2+3)^3(2x^3-5)^2$ B. $x(x^2+3)^3(2x^3-5)^2(17x^3+27x)$ C. $8x(x^2+3)^3 + 18x^2(2x^3-5)^2$
D. $(x^2+3)^3(2x^3-5)^2(8x^3+3x^2-8)$ E. $2x(x^2+3)^3(2x^3-5)^2(17x^3+27x-20)$

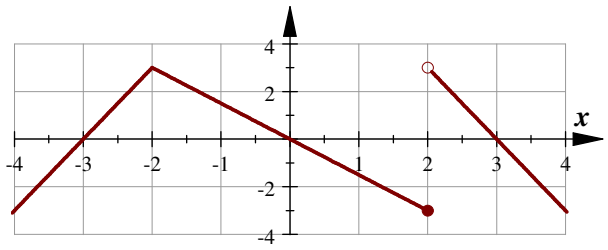
8. Given that f is continuous on $[3,9]$, $\int_3^9 f(x)\,dx = 10$, and $\int_6^9 f(x)\,dx = 2$, find $\int_3^6 2f(x)\,dx$.

A. 8 B. 16 C. 20 D. 24 E. -4

9. If $g'(x) = \cos x + b$ for some fixed constant b, then a possible formula for $g(x)$ is

A. $-\sin x$ B. $\sin x + bx$ C. $\sin x$ D. $-\sin x + bx$ E. $-\sin x + b$

10. Give the value(s) of x where the function below is **NOT** differentiable.



A. $x = 2$ B. $x = -2, 2$ C. $x = -3, 0, 3$ D. $x = -2, 0, 2$

E. The function is differentiable everywhere

11. The amount, A, of radioactive material present (in grams) after t years is given by the equation $A(t) = 3000e^{-0.02t}$. At what rate is the amount of radioactive material decreasing after 20 years (round to the nearest tenth of a gram per year)?

A. 21.6 grams/yr B. 40.2 grams/yr C. 80.4 grams/yr D. 2,011.0 grams/yr E. 3,261.9 grams/yr

12. Use the chain rule to find $\frac{dy}{dx}$ where $y = \frac{u-1}{u+1}$ and $u = \sqrt{x}$.

A. $\frac{1}{\sqrt{x}(1+\sqrt{x})^2}$ B. $\frac{1}{(1+\sqrt{x})^2}$ C. $\frac{1}{2\sqrt{x}}$ D. $\frac{\sqrt{x}}{(1+\sqrt{x})^2}$ E. $\frac{1-2\sqrt{x}}{1+2\sqrt{x}}$

13. Find a and b such that

$$h(x) = \begin{cases} ax^3 & \text{if } x \le 2 \\ x^2 + b & \text{if } x > 2 \end{cases}$$

is differentiable everywhere

A. $a = 1,\ b = 4$ B. $a = -1,\ b = -12$ C. $a = \frac{1}{3},\ b = -\frac{4}{3}$

D. $a = -\frac{1}{3},\ b = \frac{4}{3}$ E. $a = -2,\ b = -20$

14. A particle travels in linear motion and its velocity is obtained at equal time intervals. The table below indicates the particle's velocity, v, in cm/sec t seconds after data collection begins.

t	0	10	20	30
v	5	7	11	17

This data tells us that the particle's displacement after the first 30 seconds is

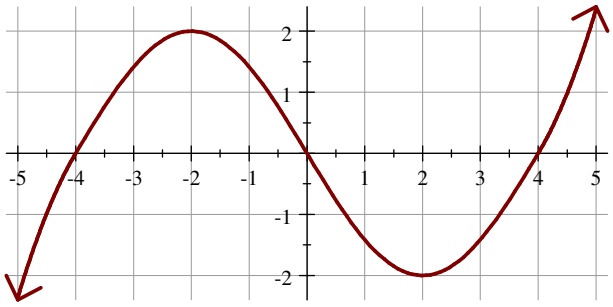
A. less than 230 cm B. more than 800 cm C. between 450 and 520 cm

D. between 520 and 800 cm E. between 230 cm and 350 cm

15. Let $y = x\sin x$. Find y''

A. $x\cos x$ B. $-x\sin x + 2\cos x$ C. $\sin x + x\cos x$ D. $2x\cos x - \sin x$ E. $-\sin x$

16. Find the interval(s) where the function f is concave upward using the graph of its derivative below.



Graph of f'

A. $(0,\infty)$ B. $(-2,2)$ C. $(-4,0)\cup(4,\infty)$ D. $(-\infty,-2)\cup(2,\infty)$ E. $(-\infty,0)$

17. A stone is dropped into a pond, creating a ripple that travels outward at a speed of 30 cm/s. Find the rate (in cm^2/s) at which the area within the circle is increasing after 5 seconds.

A. 60π B. 300π C. 4500π D. 9000π E. 22500π

18. Evaluate the limit $\lim\limits_{x\to c}\left(\dfrac{x^2-c^2}{x^4-c^4}\right)$

A. 0 B. ∞ C. $\dfrac{1}{2c^2}$ D. $\dfrac{1}{c^4}$ E. $2c^4$

19. What is the slope of the line tangent to the curve $x^2y+y^4=5$ at the point $(2,1)$?

A. $-\frac{1}{2}$ B. 0 C. 2 D. $\frac{5}{2}$ E. Cannot be determined

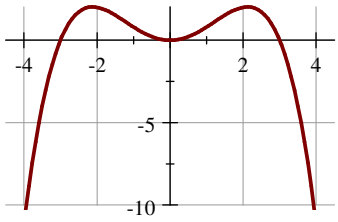
20. Find the derivative of the function $h(x)=\int_{x^2}^{0} t^3\cos t\,dt$

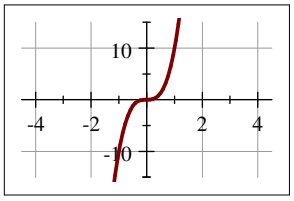
A. $x^3\cos x$ B. $x^6\cos x^2$ C. $-3x^2\sin x^2$ D. $-2x^7\cos x^2$ E. $3x^2\cos x-x^3\sin x$

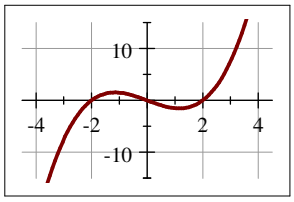
21. Find the differential dy for $y=5x^2+7x-4$.

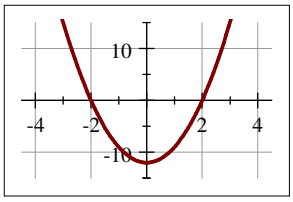
A. $17x\,dx$ B. $(10x+7)\,dx$ C. $\left(\frac{5}{3}x^3+\frac{7}{2}x^2-4x\right)dx$ D. $(10x+3)\,dx$ E. $-140x^3\,dx$

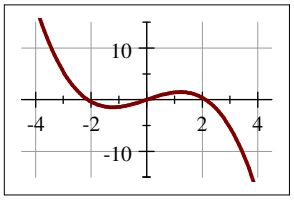
22. Match the function shown below with its derivative

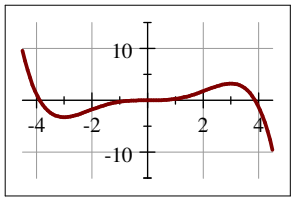


A. 

B. 

C. 

D. 

E. 

23. An object undergoing linear motion is moving forward with a constant acceleration of 3 ft/sec^2. Find the object's position after 1 minute, given that the initial velocity was 30 ft/sec and the initial position was 2 ft.

A. 31.5 ft B. 33.5 ft C. 1892 ft D. 7202 ft E. 109,800 ft

24. $\lim_{x \to 0+} \log(x) =$

A. $-\infty$ B. ∞ C. 0 D. 1 E. 10

25. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + y^2 - 3x + 6y = 9$

A. $\frac{3 - 2x}{2y + 6}$ B. $\frac{2x - 3}{2y + 6}$ C. $\frac{3 - 2x - 2y}{6}$ D. $\frac{12 - 2x}{2y + 6}$ E. $-\frac{3}{2} - x$