

TMATYC - Calculus A Test - 2014

1. For the function

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x < -2 \\ x + 3 & \text{if } -2 \leq x \leq 4 \\ -x^2 + 20 & \text{if } x > 4 \end{cases}$$

find $\lim_{x \rightarrow -2} f(x)$

- A. -4 B. -1 C. 1 D. 16 E. Does not exist

2. On what interval(s) is $f(x) = \frac{x-7}{x^2-49}$ continuous?

- A. $(-7, 7)$ B. $(-\infty, -7) \cup (-7, 7) \cup (7, \infty)$ C. $(-7, 7) \cup (7, \infty)$
 D. $(-\infty, 7) \cup (7, \infty)$ E. $(-\infty, -7) \cup (-7, \infty)$

3. Find the average value of $f(x) = x^3 + 2x - 1$ over the interval $[2, 4]$.

- A. 36 B. 30 C. 35 D. 32 E. 41

4. Find $\frac{ds}{dt}$ for $s = \frac{t^2 + 2}{3 - t^2}$

- A. $\frac{2t}{(3 - t^2)^2}$ B. $\frac{10t}{(3 - t^2)^2}$ C. $\frac{-4t^3 + 2t}{(3 - t)^2}$ D. -1 E. 0

5. Find the equation of the line tangent to the graph of $y = 3e^x - 1$ at the point $(0, 2)$.

- A. $y = 3x$ B. $y = 3e^x + 2$ C. $y = 3x - 6$ D. $y = 3x + 2$ E. $y = 2x + 2$

6. Find the vertical and horizontal asymptotes of the function

$$f(x) = \frac{x-3}{x^2 - 2x - 8}$$

- A. Vertical: $x = -2, x = 4$ Horizontal: $y = 0$ B. Vertical: $x = 0$ Horizontal: $y = -2, y = 4$
 C. Vertical: $x = 3, x = -2, x = 4$ Horizontal: $y = 0$ D. Vertical: $x = -4, x = 2$ Horizontal: none
 E. Vertical: $x = -2, x = 4$ Horizontal: none

7. Find y' if $y = (x^2 + 3)^4(2x^3 - 5)^3$

- A. $12(x^2 + 3)^3(2x^3 - 5)^2$ B. $x(x^2 + 3)^3(2x^3 - 5)^2(17x^3 + 27x)$ C. $8x(x^2 + 3)^3 + 18x^2(2x^3 - 5)^2$
 D. $(x^2 + 3)^3(2x^3 - 5)^2(8x^3 + 3x^2 - 8)$ E. $2x(x^2 + 3)^3(2x^3 - 5)^2(17x^3 + 27x - 20)$

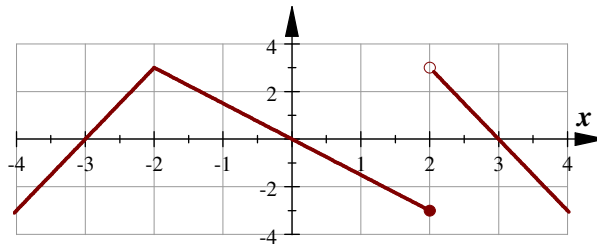
8. Given that f is continuous on $[3, 9]$, $\int_3^9 f(x) dx = 10$, and $\int_6^9 f(x) dx = 2$, find $\int_3^6 2f(x) dx$.

- A. 8 B. 16 C. 20 D. 24 E. -4

9. If $g'(x) = \cos x + b$ for some fixed constant b , then a possible formula for $g(x)$ is

- A. $-\sin x$ B. $\sin x + bx$ C. $\sin x$ D. $-\sin x + bx$ E. $-\sin x + b$

10. Give the value(s) of x where the function below is **NOT** differentiable.



- A. $x = 2$ B. $x = -2, 2$ C. $x = -3, 0, 3$ D. $x = -2, 0, 2$

E. The function is differentiable everywhere

11. The amount, A , of radioactive material present (in grams) after t years is given by the equation $A(t) = 3000e^{-0.02t}$. At what rate is the amount of radioactive material decreasing after 20 years (round to the nearest tenth of a gram per year)?

- A. 21.6 grams/yr B. 40.2 grams/yr C. 80.4 grams/yr D. 2,011.0 grams/yr E. 3,261.9 grams/yr

12. Use the chain rule to find $\frac{dy}{dx}$ where $y = \frac{u-1}{u+1}$ and $u = \sqrt{x}$.

- A. $\frac{1}{\sqrt{x}(1+\sqrt{x})^2}$ B. $\frac{1}{(1+\sqrt{x})^2}$ C. $\frac{1}{2\sqrt{x}}$ D. $\frac{\sqrt{x}}{(1+\sqrt{x})^2}$ E. $\frac{1-2\sqrt{x}}{1+2\sqrt{x}}$

13. Find a and b such that

$$h(x) = \begin{cases} ax^3 & \text{if } x \leq 2 \\ x^2 + b & \text{if } x > 2 \end{cases}$$

is differentiable everywhere

- A. $a = 1, b = 4$ B. $a = -1, b = -12$ C. $a = \frac{1}{3}, b = -\frac{4}{3}$
 D. $a = -\frac{1}{3}, b = \frac{4}{3}$ E. $a = -2, b = -20$

14. A particle travels in linear motion and its velocity is obtained at equal time intervals. The table below indicates the particle's velocity, v , in cm/sec t seconds after data collection begins.

t	0	10	20	30
v	5	7	11	17

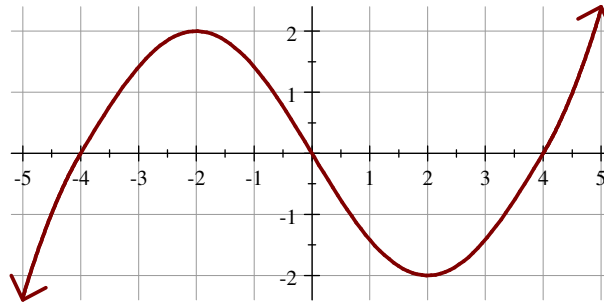
This data tells us that the particle's displacement after the first 30 seconds is

- A. less than 230 cm B. more than 800 cm C. between 450 and 520 cm
 D. between 520 and 800 cm E. between 230 cm and 350 cm

15. Let $y = x \sin x$. Find y''

- A. $x \cos x$ B. $-x \sin x + 2 \cos x$ C. $\sin x + x \cos x$ D. $2x \cos x - \sin x$ E. $-\sin x$

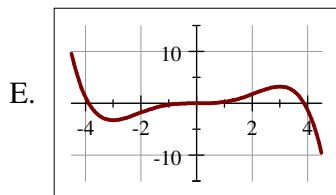
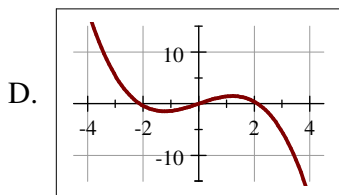
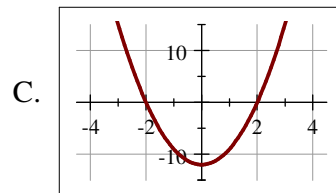
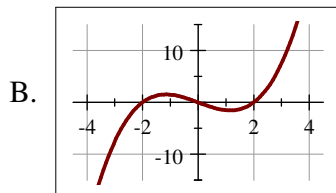
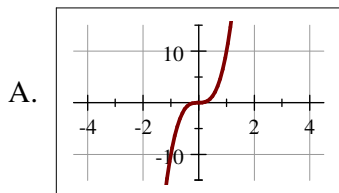
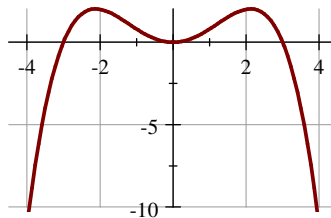
16. Find the interval(s) where the function f is concave upward using the graph of its derivative below.



Graph of f'

- A. $(0, \infty)$ B. $(-2, 2)$ C. $(-4, 0) \cup (4, \infty)$ D. $(-\infty, -2) \cup (2, \infty)$ E. $(-\infty, 0)$
17. A stone is dropped into a pond, creating a ripple that travels outward at a speed of 30 cm/s. Find the rate (in cm^2/s) at which the area within the circle is increasing after 5 seconds.
- A. 60π B. 300π C. 4500π D. 9000π E. 22500π
18. Evaluate the limit $\lim_{x \rightarrow c} \left(\frac{x^2 - c^2}{x^4 - c^4} \right)$
- A. 0 B. ∞ C. $\frac{1}{2c^2}$ D. $\frac{1}{c^4}$ E. $2c^4$
19. What is the slope of the line tangent to the curve $x^2y + y^4 = 5$ at the point $(2, 1)$?
- A. $-\frac{1}{2}$ B. 0 C. 2 D. $\frac{5}{2}$ E. Cannot be determined
20. Find the derivative of the function $h(x) = \int_{x^2}^0 t^3 \cos t \, dt$
- A. $x^3 \cos x$ B. $x^6 \cos x^2$ C. $-3x^2 \sin x^2$ D. $-2x^7 \cos x^2$ E. $3x^2 \cos x - x^3 \sin x$
21. Find the differential dy for $y = 5x^2 + 7x - 4$.
- A. $17x \, dx$ B. $(10x + 7) \, dx$ C. $\left(\frac{5}{3}x^3 + \frac{7}{2}x^2 - 4x \right) \, dx$ D. $(10x + 3) \, dx$ E. $-140x^3 \, dx$

22. Match the function shown below with its derivative



23. An object undergoing linear motion is moving forward with a constant acceleration of 3 ft/sec^2 . Find the object's position after 1 minute, given that the initial velocity was 30 ft/sec and the initial position was 2 ft .

- A. 31.5 ft B. 33.5 ft C. 1892 ft D. 7202 ft E. 109,800 ft

24. $\lim_{x \rightarrow 0^+} \log(x) =$

- A. $-\infty$ B. ∞ C. 0 D. 1 E. 10

25. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2 + y^2 - 3x + 6y = 9$

- A. $\frac{3-2x}{2y+6}$ B. $\frac{2x-3}{2y+6}$ C. $\frac{3-2x-2y}{6}$ D. $\frac{12-2x}{2y+6}$ E. $-\frac{3}{2} - x$