TMATYC - Calculus A Test - 2015

1. If $y = \ln(\ln(4x^2))$ then $\dfrac{dy}{dx} =$

A. $\dfrac{1}{\ln(4x^2)}$ B. $\dfrac{1}{8x}$ C. $\dfrac{2}{x}$ D. $\dfrac{1}{\ln(\ln(4x^2))}$ E. $\dfrac{2}{x\ln(4x^2)}$

2. If $\sinh x = \dfrac{e^x - e^{-x}}{2}$, then the expression $7\sinh(\ln x)$ can be expressed as

A. $\dfrac{7}{2}\left(x - \dfrac{1}{x}\right)$ B. $x - \dfrac{1}{x}$ C. $\dfrac{2x}{7(x^2-1)}$ D. $\dfrac{7(x^2-1)}{2x}$ E. $7x$

3. $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{x}$

A. 1 B. 0 C. -1 D. ∞ E. Does not exist

4. Given that $\int_0^c \pi x\,dx = \pi^3$, the value of c is

A. $\sqrt{2}\,\pi$ B. $\dfrac{\pi}{\sqrt{2}}$ C. 3π D. π E. π^2

5. The domain of $h(x) = \sqrt{x+20} - \sqrt{30-x}$ is

A. $[0,\infty)$ B. $(-\infty,20]\cup[30,\infty)$ C. $(-20,30)$ D. $[-20,30]$ E. $(-\infty,\infty)$

6. If $f(x) = (x^2+2)^3$, then $f'(x) =$

A. $12x^2$ B. $2x$ C. $3(x^2+2)$ D. $3(x^2+2)^2$ E. $6x(x^2+2)^2$

7. If $y = x^{\ln x}$, then $\dfrac{dy}{dx} =$

A. $\dfrac{(\ln x)^2}{x}$ B. $\dfrac{2\ln x}{x}$ C. $(\ln x)x^{\ln x - 1}$ D. $(\ln x)x^{\ln x}$ E. $2(\ln x)x^{\ln x - 1}$

8. The equation of the tangent line to the given function $f(x) = 2^x$ at $x = -1$ is given by

A. $y = \left(\frac{1}{2}\ln 2\right)x + 2\ln 2 - 1$ B. $y = \frac{1}{2}x\ln 2 + 1 - \frac{1}{2}\ln 2$ C. $y = -\frac{1}{2}x\ln 2 - 2\ln 2 + \frac{1}{2}$

D. $y = \frac{1}{2}x\ln 2 + \frac{1}{2}\ln 2 + \frac{1}{2}$ E. $y = \frac{1}{2}x + 1$

9. The critical values of the function $g(x) = e^{2x} - 10e^x + 8x + 12$

A. $\{1,4\}$ B. $\{0,\ln 4\}$ C. $\{-1,4\}$ D. $\{0,-\ln 4\}$ E. There are no critical values

10. Let a, b, and c be real constants such that $abc > 0$. What value of x will minimize the following expression

$$a + b + c + x - 4(abcx)^{1/4}$$

A. $x = \dfrac{a+b+c}{abc}$ B. $x = \dfrac{-a-b-c}{(abc)^{3/4}}$ C. $x = (abc)^{1/4}$ D. $x = (abc)^{1/3}$ E. $x = 0$

11. If $y = \sqrt{1-x^2}\,\sin^{-1}x$, then

A. $(1-x^2)\dfrac{dy}{dx} - xy = 1-x^2$ B. $(1-x^2)\dfrac{dy}{dx} + xy = 1-x^2$ C. $(1+x^2)\dfrac{dy}{dx} - xy = 1-x^2$

D. $(1+x^2)\dfrac{dy}{dx} + xy = 1-x^2$ E. $(1+x^2)\dfrac{dy}{dx} + xy = 1+x^2$

12. $\lim\limits_{t\to 0}\dfrac{a^t - b^t}{t} =$

A. $\ln\left(\dfrac{b}{a}\right)$ B. $\ln\left(\dfrac{a}{b}\right)$ C. 0 D. $a-b$ E. Does not exist

13. The area of a circle is changing at a rate of 5 cm^2/s. The instant the circle's area is 100 cm^2, the rate at which its radius is changing with respect to time is

A. $\frac{\sqrt{\pi}}{4}$ cm/s B. $\frac{4}{\sqrt{\pi}}$ cm/s C. $\sqrt{4\pi}$ cm/s D. $\frac{1}{4\sqrt{\pi}}$ cm/s E. $\frac{5}{2\pi}$ cm/s

14. If $y = xe^{x^2-3x}$, then y decreases over the interval

A. $\left(-\infty, \frac{1}{2}\right)$ B. $\left(\frac{1}{2}, 1\right)$ C. $(1,\infty)$ D. $\left(0, \frac{3}{2}\right)$ E. $(-\infty,\infty)$

15. The area bounded between $ay = 2x^2$ and $y^2 = 4ax$ is

A. $\dfrac{2}{3}a^2$ B. $\dfrac{3}{2}a^2$ C. $\dfrac{a^2}{6}$ D. $\dfrac{a^2}{3}$ E. $\dfrac{1}{8a^3}$

16. Find $\int_{\pi/2}^{\pi}\left[\cos(2x) + \sin\left(\frac{1}{2}x\right)\right]dx$

A. $2\sqrt{2}$ B. $\dfrac{\sqrt{2}}{4}$ C. $\sqrt{2}$ D. 4 E. $\dfrac{\sqrt{2}}{2}$

17. Find $\int \sin^2\theta\, d\theta$

A. $-\cos^2\theta + C$ B. $2\sin\theta\cos\theta + C$ C. $\frac{1}{2}\theta + \frac{1}{2}\sin 2\theta + C$

D. $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$ E. $\frac{1}{4}(\theta - \sin 2\theta) + C$

18. How many inflection points are there on the graph of the function $f(x) = x^4 + 4x^3 + 6x^2 + 7$?

A. 0 B. 1 C. 2 D. 3 E. 4

19. Find $F'(x)$ given that $F(x) = \tan x \ln(3x)$

A. $\sec x\left[\dfrac{\sin x}{x} + \dfrac{\ln(3x)}{\cos x}\right]$ B. $\dfrac{\tan x}{3x} + 3\ln(3x)\sec^2 x$ C. $\dfrac{\tan x}{3} + \ln(3x)\sec^2 x$

D. $\dfrac{\sec^2 x}{3x} + \sin x \ln(3x)$ E. $\dfrac{\sec^2 x}{x}$

20. Evaluate $\int \dfrac{\sinh x + \cos x}{\sin x + \cosh x}\,dx$

A. $\ln|\sin x + \cosh x| + C$ B. $-\ln|\sin x + \cosh x| + C$ C. $\ln|\sinh x + \cos x| + C$

D. $\sin x + \cosh x + C$ E. $\dfrac{\cosh x + \sin x}{-\cos x + \sinh x} + C$

21. Simplify the expression $\dfrac{1}{\log_a(abc)} + \dfrac{1}{\log_b(abc)} + \dfrac{1}{\log_c(abc)}$

A. 3 B. 1 C. 0 D. −1 E. −3

22. If $f(x) = \int_1^{x^2-\sin x} \frac{1}{t}\,dt$, then find $f'(x)$

A. $\ln|x^2 - \sin x|$ B. $\dfrac{2x - \cos x}{x^2 - \sin x}$ C. $\dfrac{1}{x^2 - \sin x}$ D. $(2x - \cos x)\ln|x^2 - \sin x|$ E. $\ln|2x - \cos x|$

23. If f is continuous and $\int_0^4 f(x)\,dx = 10$, find $\int_0^2 f(2x)\,dx$

A. 10 B. 20 C. 5 D. 4 E. 2.5

24. If $x^3 + 3x^2y + y^3 = 8$ find $\dfrac{dy}{dx}$

A. $3x^2 + 6xy + 3y^2$ B. $3x^2 + 6xy$ C. $\dfrac{3x^2}{6x + 3y^2}$ D. $-\dfrac{x^2 + 2xy}{x^2 + y^2}$ E. $\dfrac{3x}{x - y}$

25. Find the equation of the curve $y = f(x)$, given that $\dfrac{d^2y}{dx^2} = 6x - 4$ and that the curve passes through the point $(1,3)$ at which the slope of the tangent line is −5.

A. $y = x^3 + 2x^2 - 4x + 4$ B. $y = 2x^3 - 2x^2 - 4x + 7$ C. $y = x^3 - 2x^2 + 4x + 2$

D. $y = x^3 - 2x^2 - 4x + 8$ E. $y = x^6 - 2x^2 - 5x + 9$