

TMATYC - Calculus A Test - 2015

1. If $y = \ln(\ln(4x^2))$ then $\frac{dy}{dx} =$
 A. $\frac{1}{\ln(4x^2)}$ B. $\frac{1}{8x}$ C. $\frac{2}{x}$ D. $\frac{1}{\ln(\ln(4x^2))}$ E. $\frac{2}{x\ln(4x^2)}$
2. If $\sinh x = \frac{e^x - e^{-x}}{2}$, then the expression $7 \sinh(\ln x)$ can be expressed as
 A. $\frac{7}{2} \left(x - \frac{1}{x}\right)$ B. $x - \frac{1}{x}$ C. $\frac{2x}{7(x^2 - 1)}$ D. $\frac{7(x^2 - 1)}{2x}$ E. $7x$
3. $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$
 A. 1 B. 0 C. -1 D. ∞ E. Does not exist
4. Given that $\int_0^c \pi x dx = \pi^3$, the value of c is
 A. $\sqrt{2} \pi$ B. $\frac{\pi}{\sqrt{2}}$ C. 3π D. π E. π^2
5. The domain of $h(x) = \sqrt{x+20} - \sqrt{30-x}$ is
 A. $[0, \infty)$ B. $(-\infty, 20] \cup [30, \infty)$ C. $(-20, 30)$ D. $[-20, 30]$ E. $(-\infty, \infty)$
6. If $f(x) = (x^2 + 2)^3$, then $f'(x) =$
 A. $12x^2$ B. $2x$ C. $3(x^2 + 2)$ D. $3(x^2 + 2)^2$ E. $6x(x^2 + 2)^2$
7. If $y = x^{\ln x}$, then $\frac{dy}{dx} =$
 A. $\frac{(\ln x)^2}{x}$ B. $\frac{2 \ln x}{x}$ C. $(\ln x)x^{\ln x - 1}$ D. $(\ln x)x^{\ln x}$ E. $2(\ln x)x^{\ln x - 1}$
8. The equation of the tangent line to the given function $f(x) = 2^x$ at $x = -1$ is given by
 A. $y = \left(\frac{1}{2} \ln 2\right)x + 2 \ln 2 - 1$ B. $y = \frac{1}{2}x \ln 2 + 1 - \frac{1}{2} \ln 2$ C. $y = -\frac{1}{2}x \ln 2 - 2 \ln 2 + \frac{1}{2}$
 D. $y = \frac{1}{2}x \ln 2 + \frac{1}{2} \ln 2 + \frac{1}{2}$ E. $y = \frac{1}{2}x + 1$
9. The critical values of the function $g(x) = e^{2x} - 10e^x + 8x + 12$
 A. $\{1, 4\}$ B. $\{0, \ln 4\}$ C. $\{-1, 4\}$ D. $\{0, -\ln 4\}$ E. There are no critical values
10. Let a , b , and c be real constants such that $abc > 0$. What value of x will minimize the following expression

$$a + b + c + x - 4(abcx)^{1/4}$$
 A. $x = \frac{a+b+c}{abc}$ B. $x = \frac{-a-b-c}{(abc)^{3/4}}$ C. $x = (abc)^{1/4}$ D. $x = (abc)^{1/3}$ E. $x = 0$

11. If $y = \sqrt{1-x^2} \sin^{-1}x$, then

- A. $(1-x^2) \frac{dy}{dx} - xy = 1-x^2$ B. $(1-x^2) \frac{dy}{dx} + xy = 1-x^2$ C. $(1+x^2) \frac{dy}{dx} - xy = 1-x^2$
D. $(1+x^2) \frac{dy}{dx} + xy = 1-x^2$ E. $(1+x^2) \frac{dy}{dx} + xy = 1+x^2$

12. $\lim_{t \rightarrow 0} \frac{a^t - b^t}{t} =$

- A. $\ln\left(\frac{b}{a}\right)$ B. $\ln\left(\frac{a}{b}\right)$ C. 0 D. $a-b$ E. Does not exist

13. The area of a circle is changing at a rate of $5 \text{ cm}^2/\text{s}$. The instant the circle's area is 100 cm^2 , the rate at which its radius is changing with respect to time is

- A. $\frac{\sqrt{\pi}}{4} \text{ cm/s}$ B. $\frac{4}{\sqrt{\pi}} \text{ cm/s}$ C. $\sqrt{4\pi} \text{ cm/s}$ D. $\frac{1}{4\sqrt{\pi}} \text{ cm/s}$ E. $\frac{5}{2\pi} \text{ cm/s}$

14. If $y = xe^{x^2-3x}$, then y decreases over the interval

- A. $(-\infty, \frac{1}{2})$ B. $(\frac{1}{2}, 1)$ C. $(1, \infty)$ D. $(0, \frac{3}{2})$ E. $(-\infty, \infty)$

15. The area bounded between $ay = 2x^2$ and $y^2 = 4ax$ is

- A. $\frac{2}{3}a^2$ B. $\frac{3}{2}a^2$ C. $\frac{a^2}{6}$ D. $\frac{a^2}{3}$ E. $\frac{1}{8a^3}$

16. Find $\int_{\pi/2}^{\pi} [\cos(2x) + \sin(\frac{1}{2}x)] dx$

- A. $2\sqrt{2}$ B. $\frac{\sqrt{2}}{4}$ C. $\sqrt{2}$ D. 4 E. $\frac{\sqrt{2}}{2}$

17. Find $\int \sin^2\theta d\theta$

- A. $-\cos^2\theta + C$ B. $2\sin\theta\cos\theta + C$ C. $\frac{1}{2}\theta + \frac{1}{2}\sin 2\theta + C$
D. $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$ E. $\frac{1}{4}(\theta - \sin 2\theta) + C$

18. How many inflection points are there on the graph of the function $f(x) = x^4 + 4x^3 + 6x^2 + 7$?

- A. 0 B. 1 C. 2 D. 3 E. 4

19. Find $F'(x)$ given that $F(x) = \tan x \ln(3x)$

- A. $\sec x \left[\frac{\sin x}{x} + \frac{\ln(3x)}{\cos x} \right]$ B. $\frac{\tan x}{3x} + 3 \ln(3x) \sec^2 x$ C. $\frac{\tan x}{3} + \ln(3x) \sec^2 x$
D. $\frac{\sec^2 x}{3x} + \sin x \ln(3x)$ E. $\frac{\sec^2 x}{x}$

20. Evaluate $\int \frac{\sinh x + \cos x}{\sin x + \cosh x} dx$

- A. $\ln |\sin x + \cosh x| + C$ B. $-\ln |\sin x + \cosh x| + C$ C. $\ln |\sinh x + \cos x| + C$
D. $\sin x + \cosh x + C$ E. $\frac{\cosh x + \sin x}{-\cos x + \sinh x} + C$

21. Simplify the expression $\frac{1}{\log_a(abc)} + \frac{1}{\log_b(abc)} + \frac{1}{\log_c(abc)}$
- A. 3 B. 1 C. 0 D. -1 E. -3
22. If $f(x) = \int_1^{x^2 - \sin x} \frac{1}{t} dt$, then find $f'(x)$
- A. $\ln|x^2 - \sin x|$ B. $\frac{2x - \cos x}{x^2 - \sin x}$ C. $\frac{1}{x^2 - \sin x}$ D. $(2x - \cos x) \ln|x^2 - \sin x|$ E. $\ln|2x - \cos x|$
23. If f is continuous and $\int_0^4 f(x) dx = 10$, find $\int_0^2 f(2x) dx$
- A. 10 B. 20 C. 5 D. 4 E. 2.5
24. If $x^3 + 3x^2y + y^3 = 8$ find $\frac{dy}{dx}$
- A. $3x^2 + 6xy + 3y^2$ B. $3x^2 + 6xy$ C. $\frac{3x^2}{6x + 3y^2}$ D. $-\frac{x^2 + 2xy}{x^2 + y^2}$ E. $\frac{3x}{x - y}$
25. Find the equation of the curve $y = f(x)$, given that $\frac{d^2y}{dx^2} = 6x - 4$ and that the curve passes through the point $(1, 3)$ at which the slope of the tangent line is -5 .
- A. $y = x^3 + 2x^2 - 4x + 4$ B. $y = 2x^3 - 2x^2 - 4x + 7$ C. $y = x^3 - 2x^2 + 4x + 2$
D. $y = x^3 - 2x^2 - 4x + 8$ E. $y = x^6 - 2x^2 - 5x + 9$