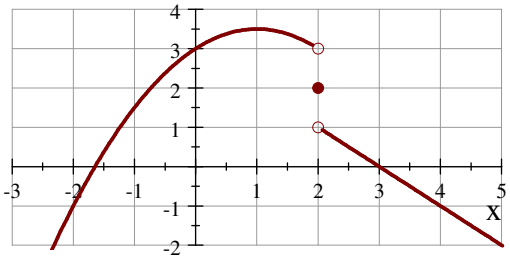
TMATYC - Calculus A Test - 2016

1. Use the graph of the function g shown below to determine $\lim_{x\to 2} g(x)$



A. 0 B. 1 C. 2 D. 3 E. the limit does not exist

2. The signum function, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

Find $\lim_{x\to 0} |\operatorname{sgn} x|$

A. -1 B. 0 C. 0.5 D. 1 E. the limit does not exist

3. Find all the values of a such that the following limit exists: $\lim_{x\to 3} \dfrac{6x^2 + (6a+b)x + ab}{2x^2 - x - 15}$

A. $-\dfrac{5}{2}$ B. $-5, 3$ C. $-\dfrac{15}{2}, \dfrac{5}{2}$ D. -3 E. there are no such values of a

4. If $k > 1$, over what interval(s) is the function $h(x) = \dfrac{\ln(x^2 - k)}{x - k}$ continuous?

A. $\left(-\infty, -\sqrt{k}\right) \cup \left(\sqrt{k}, k\right) \cup (k, \infty)$ B. $\left(\sqrt{k}, k\right) \cup (k, \infty)$ C. $(-\infty, k) \cup (k, \infty)$

D. $\left(0, -\sqrt{k}\right) \cup \left(\sqrt{k}, k\right) \cup (k, \infty)$ E. $(-\infty, \infty)$

5. Which statement is true?

A. If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a} [f(x) + g(x)]$ does not exist.

B. If $f(a) = L$ then $\lim_{x\to a} f(x) = L$.

C. If f is continuous at $x = a$, then f is differentiable at $x = a$.

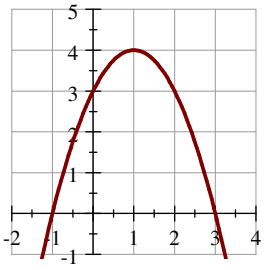
D. If f is differentiable at $x = a$, then so is $|f|$.

E. If f is differentiable at $x = a$, then f is continuous at $x = a$.

6. If $f(x) = x^2 + \sin x + 2$, then $\lim_{x\to \pi} \dfrac{f(x) - f(\pi)}{x - \pi} =$

A. $\pi^2 + 2$ B. $2\pi - 1$ C. $2\pi + 1$ D. 0 E. the limit does not exist

7. The graph of the function h is shown below.



If A is the average rate of change of h over the interval $[0,3]$, then the best estimate for the expression $\dfrac{h'(2)}{A}$ is

A. -3 B. -1 C. $-\dfrac{1}{3}$ D. 2 E. 3

8. The ceiling function, denoted $\lceil x \rceil$, is defined as the smallest integer greater than or equal to x. If $g(x) = \lceil x \rceil$, then find $g'(5)$.

A. -1 B. 0 C. 5 D. 6 E. g is not differentiable at $x = 5$

9. If $y = ae^v + \dfrac{c}{v}$ for constants a and c, find $\dfrac{dy}{dv}$

A. $ave^{v-1} + c$ B. $ae^v + \dfrac{1}{v}$ C. $ae^v - \dfrac{c}{v^2}$ D. $\dfrac{ae^v}{v} + \dfrac{2c}{v^2}$ E. $ae^v(1+v)$

10. A kite is tested in a wind tunnel. The function $h = f(s)$ gives the height, in feet, of a kite for a wind speed of s miles per hour. What are the units of $f'(s)$?

A. feet per hour B. feet per second C. feet per (miles per hour)

D. miles per hour E. (miles per hour) per foot

11. If $w = \sqrt{\ln p}$, then $\dfrac{dw}{dp} =$

A. $\dfrac{1}{2p\sqrt{\ln p}}$ B. $\dfrac{1}{2\sqrt{\ln p}}$ C. $\sqrt{\dfrac{1}{p}}$ D. $\dfrac{1}{2p}$ E. $\sqrt{\dfrac{p}{p+1}}$

12. Find the equation of the line tangent to $y = (2x+1)^5$ at $x = 0$.

A. $y = 32x + 1$ B. $y = 5(2x+1)$ C. $y = 10x + 1$ D. $y = 5x + 1$ E. $y = 2x + 1$

13. Which of the following points on the curve $y^3 - xy + x^3 = 1$ is where the tangent line has slope -1?

A. $(1,-1)$ B. $(1,1)$ C. $(-1,1)$ D. $(1,0)$ E. $(0,1)$

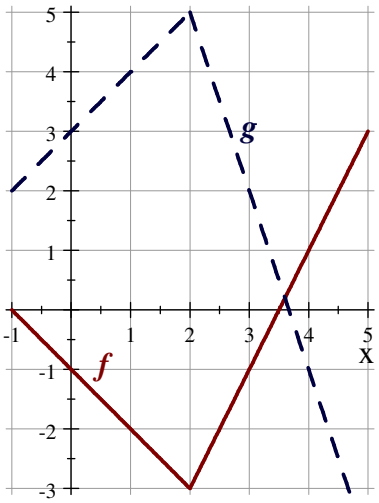
14. If $\dfrac{d}{dx}[h(2x)] = x^2 + 6$, then $h'(4) =$

A. 4 B. 5 C. 8 D. 10 E. 22

15. Find $f^{(n)}(x)$, the nth derivative of $f(x) = xe^x$.

A. e^x B. ne^x C. nxe^x D. $(n+x)e^x$ E. $x^n e^{x-n}$

16. The graphs of f (solid line) and g (dashed line) are shown below.



If $h(x) = \dfrac{f(x)}{g(x)}$, find $h'(3)$

A. $\dfrac{1}{4}$ B. $\dfrac{2}{3}$ C. $-\dfrac{2}{3}$ D. $-\dfrac{1}{3}$ E. $-\dfrac{1}{2}$

17. An airplane flies at a speed of 600 miles per hour at a constant altitude of 4 miles toward a point directly overhead of an observer on the ground. What is the rate of change of the angle of elevation from the observer to the airplane the instant the airplane is 5 miles from the observer? Express your answer in radians per hour.

A. 16 B. 96 C. 216 D. 360 E. 480

18. The volume, V, of a balloon in cubic feet is given by the function

$$V(t) = \frac{3t^2 + 1}{t + 1}$$

where t is the time in seconds since inflation of the balloon began. Find the limit as $t \to \infty$ of the rate of change of the volume of the balloon. Give your answer in cubic feet per second.

A. 3 B. 4 C. 6 D. 27 E. ∞

19. The number 30! has how many zeros at the end?

A. 0 B. 6 C. 7 D. 10 E. 23

20. Let g be continuous on the interval $[a,b]$ and let it have a maximum value of M on this same interval. Let c be the only critical point for g on the interval (a,b). Finally, let d be any real number other than c in the interval (a,b). Which of the following must be **true**?

A. $g'(c) = 0$ B. $g'(b) \neq 0$ C. $g(a) = M$ D. $g(c) \neq M$ E. $g(d) \neq M$

21. Let $p(x) = -x^2 + a$ and $q(x) = (x+1)^2 - b$ where a and b are real numbers. If the line tangent to $p(x)$ at $x = 1$ and the line tangent to $q(x)$ at $x = 1$ intersect at the point $(2,3)$, then $a + b =$

A. 2 B. 5 C. 10 D. 11 E. 13

22. If a is a positive constant, then the function $y = ax^4 + 3ax^3 + 3ax^2 + ax + 300$ is concave downward on the interval

A. $\left(-\frac{3}{2}, 0\right)$ B. $\left(-\infty, -\frac{3}{4}\right)$ C. $\left(-1, -\frac{1}{2}\right)$ D. $\left(-\frac{3}{4}, -\frac{1}{2}\right)$

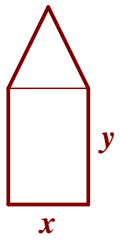
E. the function is never concave downward

23. If $f(x) = \sin^2(ax + b)$ for constants a and b, then $f''(x) =$

A. $-a^2 \sin^2(ax + b)$ B. $-(2 \sin a) \sin(ax + b)$ C. $2a^2 \cos[2(ax + b)]$

D. $-2a^2 \cos(ax + b)$ E. $2 \cos^2 a \sin^2(ax + b)$

24. A window is to be constructed in the shape of a rectangle surmounted by an equilateral triangle with base equal to the width of the rectangle as shown below. Note that x is the width and y is the height of the rectangular portion of the window.



If the perimeter of the window is 10 feet, what is the value of x (to the nearest tenth of a foot) that will maximize the surface area of the window?

A. 1.7 B. 2.3 C. 3.3 D. 4.7 E. 5.9

25. If $f'(x) = 3x^2 + 8x - 3$ and $f(0) = 2$, then $f(x) =$

A. $6x + 8$ B. $6x + 2$ C. $3x^2 + 8x + 2$ D. $x^3 + 4x^2 - 3x$ E. $x^3 + 4x^2 - 3x + 2$