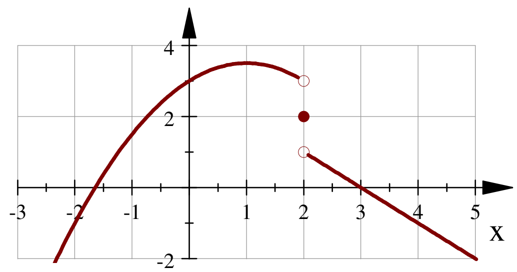
TMATYC - Calculus A Test - 2017

1. Use the graph of the function g shown below to determine $\lim_{x \to 2+} g(x)$



A. 0 B. 1 C. 2 D. 3 E. the limit does not exist

2. If $a,\ b,\ c,$ and d are positive constants, then find $\lim_{x \to \infty} \dfrac{a}{b + ce^{-dx}} =$

A. $-\infty$ B. 0 C. $\dfrac{a}{b}$ D. $\dfrac{a}{b+c}$ E. the limit does not exist

3. Let $f(x) = \dfrac{x^{-1} - 1}{x - x^2}$ and $g(x) = \dfrac{12}{x}$. If $h(x) = f\left(\frac{1}{2} g(2x)\right)$, then $\lim_{x \to 3} h(x) =$

A. 0 B. $\dfrac{1}{16}$ C. 1 D. 108 E. the limit does not exist

4. If the following function is continuous on its domain, then find the sum of m and n.

$$p(x) = \begin{cases} x^2 + mx & \text{if} \quad x < -1 \\ 5\cos(\pi x) & \text{if} \quad -1 \le x \le 2 \\ \sqrt{nx+3} & \text{if} \quad x > 2 \end{cases}$$

A. 17 B. 10 C. 5 D. $\dfrac{5}{2}$ E. 0

5. Which statement is true?

A. If $\lim_{x \to 5} \dfrac{f(x)}{g(x)}$ exists, then the limit must be $\dfrac{f(5)}{g(5)}$.

B. If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$, then $\lim_{x \to 0} [f(x) - g(x)] = 0$.

C. If f has domain $[0, \infty)$ and has no horizontal asymptote, then $\lim_{x \to \infty} f(x) = \infty$ or $\lim_{x \to \infty} f(x) = -\infty$.

D. If p is a polynomial, then $\lim_{x \to b} p(x) = p(b)$.

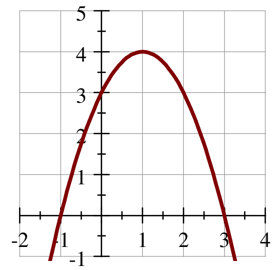
E. If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1.

6. If a is a constant and $f(x) = \sin\sqrt{ax^2 + 1}$, then $f'(x) =$

A. $\cos\sqrt{ax^2+1}$ B. $2ax\cos\sqrt{ax^2+1}$ C. $\dfrac{ax\cos\sqrt{ax^2+1}}{\sqrt{ax^2+1}}$

D. $\cos\sqrt{2ax}$ E. $\cos\sqrt{ax^2+1} + \sin\sqrt{2ax}$

7. The graph of the derivative of h is shown below.



The function h is increasing on the interval(s)

A. $(-\infty, \infty)$ B. $(-\infty, 1)$ C. $(-\infty, -1) \cup (3, \infty)$ D. $(1, \infty)$ E. $(-1, 3)$

8. If $y = (x+1)(x+2)(x+3)(x+4)(x+5)$ then $\dfrac{d^5y}{dx^5} =$

A. $5x^4$ B. $5x + 120$ C. 0 D. 1 E. 120

9. If $g(x) = |x^2 - 4x - 1|$ then $g'(1) =$

A. -2 B. 2 C. $-\frac{8}{3}$ D. $\frac{8}{3}$ E. 4

10. The concentration C, in moles per liter (mol/L), of a product in a mixture t seconds (sec) after a reaction with other reactants is given by

$$C = \frac{16kt}{4kt + 1}$$

where k is a constant. What happens to the rate of change of this concentration with respect to time as $t \to \infty$? It approaches

A. 4 mol/L B. 4 mol/(L•sec) C. 0 mol/L D. 0 mol/(L•sec) E. $4k$ mol/L

11. For differentiable function f, let $w = f(\ln(0.2x))$. Find the value of $\left.\dfrac{dw}{dx}\right|_{x=5}$ given the following values of f and f'.

x	0	$\frac{1}{5}$	1
$f(x)$	10	15	20
$f'(x)$	20	30	40

A. 4 B. 10 C. 15 D. 30 E. 40

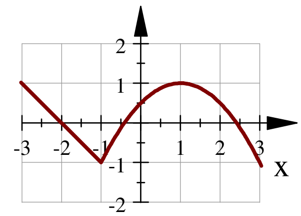
12. Find the equation of the tangent line to the curve $x^4 - 4xy + y^4 = 6$ at the point $(-1, 1)$.

A. $y = x + 2$ B. $y = -5x - 4$ C. $y = \frac{13}{8}x + \frac{21}{8}$ D. $y = 1$ E. $x = -1$

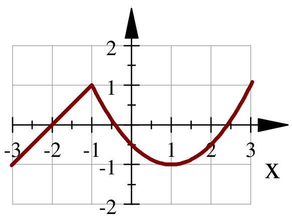
13. Find the derivative of $y = \tan^{-1}(e^{2x})$

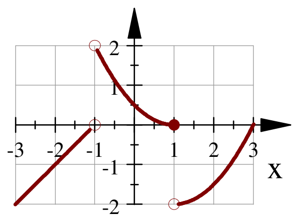
A. $2e^{2x}\sec^2(e^{2x})$ B. $2\sec^2(e^{2x})$ C. $\dfrac{2xe^x}{1+e^{4x}}$ D. $\dfrac{e^{2x}}{1+e^{4x}}$ E. $\dfrac{2e^{2x}}{1+e^{4x}}$

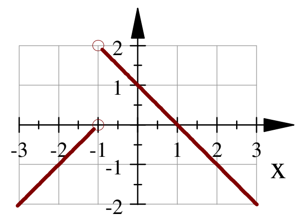
14. The graph of the function $p(x)$ is shown below.

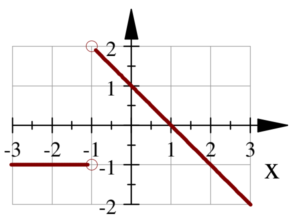


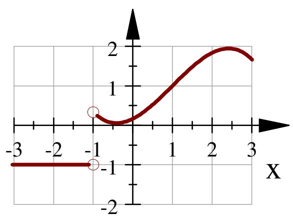
Which of the following is the graph of $p'(x)$?

A.


B.


C.


D.


E.


15. Let f be a one-to-one function. If $f(4) = 5$ and $f'(4) = \frac{2}{3}$, find $(f^{-1})'(5)$.

A. 0 B. $\frac{1}{4}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$ E. $\frac{10}{3}$

16. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing the instant the altitude is 10 cm and the area is 100 cm^2?

A. decreasing at 1.6 cm/min B. decreasing at 0.4 cm/min C. increasing at 0.4 cm/min

D. increasing at 4 cm/min E. increasing at 20 cm/min

17. Suppose that we don't have a formula for $g(x)$, but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x. Use a linear approximation to estimate $g(2.05)$. If needed, round to the nearest hundredth.

A. -3.99 B. -3.85 C. 0.68 D. 3.03 E. 4.58

18. Consider the function $h(t) = 5t(6-7t)^{1/3}$. What is the sum of the critical points for h?

A. 0 B. $\frac{6}{7}$ C. $\frac{3}{2}$ D. $\frac{129}{77}$ E. 4

19. The function $y = 5\sin(200\pi t)$ has infinitely many intervals of equal width where it is increasing. What is the length of one of these intervals?

A. 0.005 B. 0.001 C. 0.01 D. 0.025 E. 0.5

20. Over what interval(s) is the function $y = 15x^4 - 11x^3 + 3x^2 + 1$ concave upward?

A. $(-\infty, \infty)$ B. $(-0.7, 1.05)$ C. $\left(-\infty, -\frac{7}{10}\right) \cup \left(\frac{11}{10}, \infty\right)$ D. $\left(-\infty, \frac{1}{6}\right) \cup \left(\frac{1}{5}, \infty\right)$

E. y is never concave upward

21. A gardener wants to enclose a rectangular garden on one side by a brick wall that costs \$80 per linear foot and the remaining three sides by a metal fence that costs \$20 per linear foot. If the garden is to enclose 1,000 ft^2, what length (to the nearest foot) of the brick wall will minimize the cost to enclose the garden?

A. 10 B. 20 C. 25 D. 30 E. 33

22. Suppose f'' is continuous for all real numbers. If $f'(2) = 0$ and $f''(2) > 0$ then which of the following must be true?

A. f has a local maximum at $x = 2$

B. f has a local minimum at $x = 2$

C. f has an inflection point at $x = 2$

D. f has a vertical tangent line at $x = 2$

E. f has a sharp turning point or corner at $x = 2$

23. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

A. $\frac{1}{4\pi}$ B. $\frac{1}{4}$ C. $\frac{1}{\pi}$ D. 1 E. π

24. If $\lim_{x \to a} f(x) = L$ where L is a real number, which of the following must be true?

A. $f'(a)$ exists

B. $f(x)$ is continuous at $x = a$

C. $f(x)$ is defined at $x = a$

D. $f(a) = L$

E. None of these must be true

25. Let g be a continuous function on the closed interval $[0, 1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is **NOT** necessarily true?

A. There exists a number h in $[0, 1]$ such that $g(h) \geq g(x)$ for all x in $[0, 1]$.

B. For all a and b in $[0, 1]$, if $a = b$, then $g(a) = g(b)$.

C. There exists a number h in $[0, 1]$ such that $g(h) = \frac{1}{2}$.

D. There exists a number h in $[0, 1]$ such that $g(h) = \frac{3}{2}$.

E. For all h in the open interval $(0, 1)$, $\lim_{x \to h} g(x) = g(h)$.