TMATYC - Calculus A Test - 2018

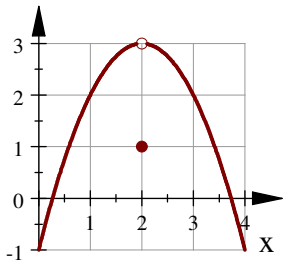
1. If $\lim_{x\to 1} f(x) = \infty$ and $\lim_{x\to 1} g(x) = 0$, then $\lim_{x\to 1}\left[\cos(g(x)) + \ln\left(1 + \frac{2}{f(x)}\right) + \arctan\left(\frac{3}{|g(x)|}\right)\right] =$

A. 0 B. 1 C. $\frac{\pi+2}{2}$ D. $e+1$ E. ∞

2. $\lim_{x\to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x-a} =$

A. $-\frac{1}{2a^{3/2}}$ B. $-\frac{1}{2\sqrt{a}}$ C. $-\frac{1}{\sqrt{a}}$ D. 0 E. the limit does not exist

3. Use the graph of the function $h(x)$ shown below to find $\lim_{x\to 2} h(x)$



A. 0 B. 1 C. 2 D. 3 E. the limit does not exist

4. If $\lim_{x\to 0} \frac{\sqrt{ax+b} - \sqrt{3}}{x} = \sqrt{3}$ then $a+b =$

A. 0 B. 3 C. $2\sqrt{3}$ D. 4 E. 9

5. If f is continuous on the interval $[a,b]$ and $f(a) < y < f(b)$, then which of the following **MUST** be true?

A. $f(b)$ must be a maximum for f on the interval $[a,b]$.

B. $f'(x)$ must be positive for all values of x in the interval (a,b).

C. $y = \frac{f(a)+f(b)}{2}$

D. There is some number c in the interval (a,b) such that $f(c) = y$.

E. There is some number c in the interval (a,b) such that $f'(c) = 0$.

6. $\lim_{h\to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} =$

A. 1 B. $3x^{1/3}$ C. $-\frac{3}{x^{1/3}}$ D. $\frac{1}{3x^{2/3}}$ E. the limit does not exist

7. Find the equation of the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3,-4)$.

A. $y = -\frac{3}{4}x - \frac{7}{4}$ B. $y = \frac{3}{4}x - \frac{49}{8}$ C. $y = \frac{3}{4}x - \frac{25}{4}$ D. $y = \frac{4}{3}x - 8$ E. $y = \frac{7}{10}x - \frac{61}{10}$

8. What is the product of all the critical points for the function $f(x) = (x-4)\sqrt[3]{2x+7}$?

A. $\frac{21}{4}$ B. $\frac{91}{16}$ C. 6 D. -14 E. $-\frac{7}{2}$

9. If $y = xe^{2x}$, then $\dfrac{d^3y}{dx^3} =$

A. $4e^{2x}(2x+3)$ B. $e^{2x}(x+3)$ C. $e^{2x}(2x+1)$ D. $8x^3e^{2x}$ E. e^{2x}

10. If $p(t) = \sin(\tan^2 t)$, then $p'(t) =$

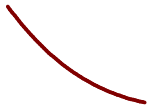
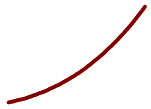
A. $\cos(\sec^4 t)$ B. $\cos(\tan^2 t)\sec^4 t$ C. $\cos(\tan^2 t) + \sin(2\tan t \sec^2 t)$

D. $2\sin(\tan t)\cos(\tan t)\sec^2 t$ E. $2\cos(\tan^2 t)\tan t \sec^2 t$

11. If a and b are constants and $g(x) = \dfrac{ax^2+b}{x^2+1}$, then $g'(1) =$

A. a B. $\frac{1}{2}(a-b)$ C. $\frac{1}{2}(a+b)$ D. $\frac{1}{4}(3a-b)$ E. $a+b$

12. If $y' > 0$ and $y'' < 0$ for all x, which of the following is a possible graph for y?

A.  B.  C.  D.  E. 

13. If the parabola $y = ax^2 + bx + c$ passes through the point $(2,6)$ and is tangent to the line $y = 4x - 11$ at $(3,1)$ then $a + b + c =$

A. 29 B. 30 C. 31 D. 33 E. 35

14. If $m(3x-5) = x^3 + 3x + 4$, then $m'(1) =$

A. -4 B. 5 C. 6 D. 8 E. 18

15. The displacement y, in feet, from equilibrium for an object on the end of a spring is given by

$$y = \frac{1}{5}\cos 3\pi t - \frac{1}{4}\sin 3\pi t$$

where t is the elapsed time in seconds. What is the velocity (in ft/s) of the object at $t = 0.5$ seconds?

A. $\frac{3\pi}{5}$ B. $\frac{3}{5}$ C. $-\frac{1}{4}$ D. $-\frac{\pi}{20}$ E. $-\frac{1}{20}$

16. A 6-foot-tall man walks at a constant speed of 3 feet per second away from a light that is 20 feet above the ground. At what speed is the length of his shadow changing (in feet per second).

A. $\frac{6}{7}$ B. $\frac{20}{7}$ C. $\frac{27}{7}$ D. 7 E. 10

17. If $f(x) = 6 - \frac{5}{x}$, find **all** the values of c in the open interval $(1,5)$ such that

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

A. ± 5 B. $\pm\sqrt{5}$ C. $\pm\frac{\sqrt{5}}{2}$ D. 1,5 E. 3

18. If a and b are positive constants with a being an even integer greater than 1, then the function $g(x) = x^a e^{-bx}$ has a local (or relative) maximum at $x =$

A. 0 B. a C. b D. $\frac{a}{b}$ E. $\frac{b}{a}$

19. Let $T = f(x)$ be a function that gives the temperature x hours after noon. For $0 \le x \le 6$, the following information is known:

$$f'(x) = 4 \text{ for } 0 \le x \le 3, \quad \text{and } f''(x) = -2 \text{ for } 3 \le x \le 6$$

When during the 6 hours after noon, is the temperature the highest?

A. $x = 0$ B. $x = 3$ C. $x = 4$ D. $x = 5$ E. $x = 6$

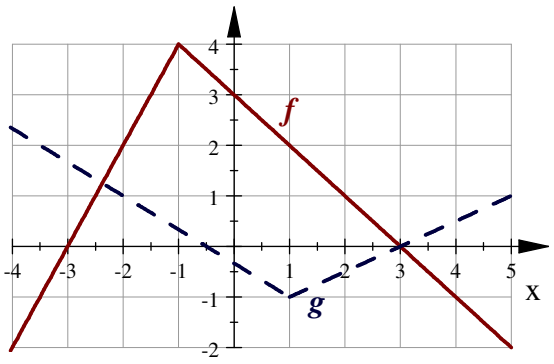
20. Which of the following intervals contain a value of x where the function $y = -\frac{1}{2}\ln(\cos 2x) - \sqrt{3}\,x + 50$ has a horizontal tangent line?

A. $(-0.2, 0.2)$ B. $(0.1, 0.3)$ C. $(0.2, 0.5)$ D. $(0.5, 0.7)$ E. $(0.7, 0.8)$

21. The position of a particle moving on the x-axis is given by $x(t) = 2t^3 - 15t^2 + 24t + 10$ where $t \ge 0$ is the elapsed time in seconds. How many units does the particle travel in the first five seconds of motion?

A. 49 B. 24 C. 21 D. 15 E. 5

Use the graphs of the functions f (solid line) and g (dashed line) shown below to answer question #'s 22 and 23.



22. If $h(x) = f(x)g(x)$ then $h'(-2) =$

A. 2 B. 5 C. -2 D. $\frac{2}{3}$ E. $-\frac{4}{3}$

23. If $p(x) = f(x^2)$ then $p'(-1) =$

A. 2 B. 4 C. -2 D. -1 E. 1

24. Two lines are tangent to the graph of $y = 4x - x^2$. If the points of tangency for these two lines occur at $x = 1$ and $x = 3$, then at what point do these two lines intersect?

A. $(1,3)$ B. $(2,-2)$ C. $(1,9)$ D. $(2,3)$ E. $(2,5)$

25. Find the equation of the tangent line to the graph of $f(x) = \dfrac{3}{x^2}$ that passes through the point $(-3,0)$ not on the graph of f.

A. $y = \frac{2}{9}x + 1$ B. $y = \frac{2}{9}x + \frac{2}{3}$ C. $y = \frac{3}{20}x + \frac{9}{20}$ D. $y = \frac{3}{2}x + \frac{9}{2}$ E. $y = \frac{3}{4}x + \frac{9}{4}$