TMATYC - Calculus B Test - 2013

1. $\sum_{k=1}^{n} \ln\left(\frac{k}{k+1}\right)^2 =$

A. $2\ln(n)$ B. $\ln(n^2+1)$ C. $-2\ln(n+1)$ D. $\ln\left(\frac{n}{n+1}\right)$ E. $\ln\left(\frac{n}{n+1}\right)^2$

2. Find the area of the region enclosed by $y = ax$ and $y = x^2$ if $a > 0$.

A. $\frac{1}{6}a^3$ B. $\frac{2}{3}a^3$ C. $\frac{5}{6}a^3$ D. $\frac{1}{2}a^2$ E. $\frac{5}{6}a^2$

3. If $y = a\sin^2(bx+c)$ then $\frac{dy}{dx} =$

A. $a\cos^2(bx+c)$ B. $ab\cos^2(bx+c)$ C. $2ab\cos(bx+c)$ D. $ab\sin[2(bx+c)]$ E. $2a\sin b$

4. Find the absolute maximum of $y = |x^2 - 2x - 3|$ on the interval $[0,4]$

A. 2 B. 3 C. 4 D. 5 E. 6

5. If $f(x) = x^2\cos x$ then $f''(\pi) =$

A. 2 B. π^2 C. $\pi^2 - 2$ D. -2π E. 0

6. If $a > 0$, then $\lim_{h\to 0} \frac{\sqrt{a^2+h} - a}{h} =$

A. 1 B. $\frac{1}{2}a^2$ C. $\frac{1}{2a}$ D. $\frac{1-2a}{2a}$ E. $\frac{a^3}{6}$

7. What is the domain of the derivative of the function $g(x) = x^{5/2}(x-2)^{2/3}$

A. All real numbers B. $[2,\infty)$ C. $(-\infty,2)\cup(2,\infty)$ D. $(0,2)\cup(2,\infty)$ E. $[0,2)\cup(2,\infty)$

8. Find the values of a and b that make the following function differentiable everywhere

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ ax+b & \text{if } x > 2 \end{cases}$$

A. $a = 0, b = 4$ B. $a = 2, b = 0$ C. $a = 4, b = -4$ D. $a = 4, b = 2$ E. $a = 2, b = 2$

9. Find the 1000^{th} derivative of $y = xe^{-x}$

A. $e^{-x}(x-1000)$ B. $e^{-x}(x+1000)$ C. e^{-x} D. $-xe^{x-1000}$ E. $e^{-x}(1+1000x)$

10. Evaluate the following integral $\int_{-1}^{2} (2x+k)\,dx$

A. $3+k$ B. $3+3k$ C. $5+k$ D. $5+3k$ E. $2+k$

11. $\lim_{x\to 1} \frac{\frac{1}{x} - 1}{\sqrt{x} - 1} =$

A. 0 B. $-\infty$ C. 1 D. -2 E. 2

12. Consider the region in the first quadrant bounded by the graphs of $y = x + 1$, $y = 0$, $x = 0$, and $x = 2$. Which one of the following integrals will give the volume of the solid generated by revolving this region about the line $y = -1$.

A. $\pi \int_0^2 (x+1)^2\,dx$ B. $\pi \int_0^2 (x+2)^2\,dx$ C. $\pi \int_0^2 \left[(x+2)^2 - 1\right] dx$

D. $2\pi \int_0^2 x(x+1)\,dx$ E. $2\pi \int_0^2 (x+1)^2\,dx$

13. $\int \frac{4x^3 + 2x}{1 + x^4}\,dx =$

A. $\frac{x^4 + x^2}{x + \frac{1}{5}x^5} + C$ B. $\frac{12x^3 + 2}{4x^3} + C$ C. $\ln\left(\frac{4x^3}{12x^3 + 2}\right) + C$

D. $\ln(1 + x^4) + C$ E. $\ln(1 + x^4) + \arctan(x^2) + C$

14. A ball is released 2 meters above the ground. Each time the ball hits the ground, it bounces back to $\frac{2}{3}$ of its previous height. What is the total distance in meters that the ball must travel before it comes to a rest on the ground?

A. 4 B. 6 C. 8 D. 10 E. 12

15. All units in a 30-unit apartment building are rented out when the monthly rent is set at \$1000. A survey shows that one unit becomes empty with each \$40 increase in rent. Additionally, each occupied unit costs the landlord \$120 in maintenance per month. What rent will maximize profit?

A. \$800 B. \$1160 C. \$1200 D. \$1400 E. \$1485

16. The base of a solid is the unit circle $x^2 + y^2 = 1$. Cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

A. $\frac{16}{3}$ B. 4 C. $\frac{25}{8}$ D. $\frac{35}{12}$ E. $\frac{4}{3}$

17. Use partial fractions to evaluate: $\int_3^8 \frac{3x}{(x+1)(x-2)}\,dx$

A. $4\ln 6$ B. $5\ln 3 - 3\ln 2$ C. $4\ln 3$ D. $4\ln 3 - \ln 2$ E. $6\ln 3 - 4\ln 2$

18. A cube-shaped block of ice with edges 20 inches long begins to melt at 8:00 a.m. Each edge decreases at a constant rate thereafter and is 8 inches long at 4:00 p.m. What was the rate of change of the block's volume at noon?

A. -7488 in^3/hr B. -882 in^3/hr C. -936 in^3/hr D. -288 in^3/hr E. -588 in^3/hr

19. Solve the differential equation $x + 2yy'\sqrt{x^2 + 1} = 0$ with $y(0) = 1$.

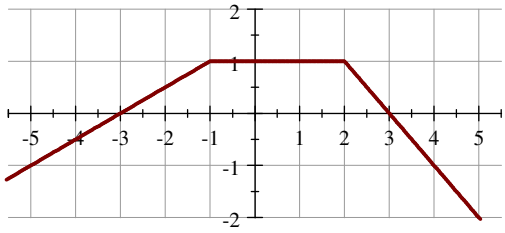
A. $\sqrt{2 - \sqrt{x^2 + 1}}$ B. $1 - \sqrt{x^2 + x}$ C. $\frac{1}{\sqrt{x^2 + 1}}$

D. $1 + \frac{2}{\sqrt{x^2 + 1}}$ E. $2 - \frac{1}{\sqrt{x^2 + 1}}$

20. Evaluate $\iint_D (x^2 + y^2)\, dA$ where D is the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

A. $\frac{14\pi}{3}$ B. $\frac{27\pi}{5}$ C. 6π D. $\frac{15\pi}{2}$ E. $\frac{255\pi}{2}$

Use the graph of the function $f(x)$ shown below to answer questions #21 and #22 below



Graph of $f(x)$

21. If $g(x) = \int_{-5}^{x} f(t)\, dt$, then $g(5) =$

A. -2 B. -1 C. $\frac{1}{2}$ D. 1 E. $\frac{3}{2}$

22. If $g(x) = \int_{-5}^{x} f(t)\, dt$, then $g'(3) =$

A. $-\frac{3}{2}$ B. -1 C. 0 D. 2 E. $\frac{7}{2}$

23. If $e^{h(x)} = 12x$ and $h(4) = \ln 48$, find $h'(4)$

A. 4 B. $\frac{3}{4}$ C. 2 D. $\frac{1}{4}$ E. 3

24. Let $g(x) = \begin{cases} \sin x & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$. Which of the following statements are true?

P: $g'(0) = 1$

Q: $g(x)$ is continuous at $x = 0$

R: $g(x)$ is differentiable at $x = 0$

A. P only B. Q only C. P and Q only D. Q and R only E. P, Q, and R

25. Suppose $S = \sum_{n=1}^{\infty} a_n$ is an infinite series with partial sum $S_n = 5 - \frac{3}{n^2}$. What is the value of a_3?

A. $\frac{\sqrt{6}}{2}$ B. $\frac{5}{12}$ C. $\frac{14}{3}$ D. $\frac{14}{9}$ E. 5