TMATYC - Calculus B Test - 2014

1. Suppose that $f(x) = ax^2 + bx + c$. Which of the following must be true?

A. f has an absolute minimum if $b < 0$ B. f has an absolute maximum if $a < 0$

C. f has an absolute maximum if $a > 0$ D.f has an absolute minimum if $a < 0$

E. $f'(x)$ is constant over the domain of f

2. Find the derivative of $f(x,y) = x^2 + xy + y^2$ at the point $(-2,-1)$ in the direction on which the function f decreases most rapidly.

A. $-3\sqrt{6}$ B. $\sqrt{41}$ C. $-\sqrt{83}$ D. $-\sqrt{41}$ E. $\sqrt{77}$

3. Find the 2014^{th} derivative of $\sin(2x)$

A. $2^{2014}\sin(2x)$ B. $-2^{2014}\sin(2x)$ C. $2^{2014}\cos(2x)$ D. $-2^{2014}\cos(2x)$ E. $2^{2014}\sin(x)$

4. Suppose that $f(t) = \sin(4t)$. Find the Laplace Transform for f:

A. $\frac{4}{s^2+16}$ B. $\frac{16}{s^2+16}$ C. $\frac{s}{s^2+16}$ D. $\frac{s}{s^2+4}$ E. $\frac{4}{s^2+4}$

5. Evaluate the following integral:

$$\int_0^{10\pi}\int_0^{15\pi} \sin(x) + \cos(y)\,dx\,dy$$

A. 0 B. 11π C. 21π D. 10π E. 20π

6. Find the equation of the plane containing the points $P(2,-3,1)$, $Q(11,4,16)$, and $R(-3,0,8)$.

A. $10x + y + 25z = 42$ B. $2x - 3y + z = 0$ C. $4x - 138y + 62z = 484$

D. $4x + 138y + 62z = -344$ E. $4x - 28y + 20z = 112$

7. Find the eigenvalue(s) for the matrix below

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 8 \end{bmatrix}$$

A. $\lambda = 0$ B. $\lambda_1 = 5 - 2\sqrt{6}$, $\lambda_2 = 5 + 2\sqrt{6}$ C. $\lambda_1 = 5 - \sqrt{6}\,i$, $\lambda_2 = 5 + \sqrt{6}\,i$

D. $\lambda_1 = -1$, $\lambda_2 = 1$ E. $\lambda_1 = 1 - 4\sqrt{2}$, $\lambda_2 = 1 + 4\sqrt{2}$

8. Find the length of the curve $\vec{r}(t) = \langle 9 + 2t^2, 2t^2 - 2, 4 - t^2 \rangle$ over the interval$1 \le t \le 4$

A. 53 B. 51 C. 135 D. 45 E. 756

9. Find $\frac{df}{dx}$ if $f(\theta) = \frac{2\sin^2(\theta)}{\theta}$

A. $\frac{4\theta\sin(\theta)\cos(\theta) - 2\sin^2(\theta)}{\theta^2}$ B. $\frac{4x\sin(x)\cos(x) - 2\sin^2(x)}{x^2}$

C. $\frac{4\theta\sin(\theta) - 2\sin^2(\theta)}{\theta^2}$ D. $\frac{4\theta\sin(\theta)\cos(\theta) + 2\sin^2(\theta)}{\theta^2}$ E. 0

10. Suppose that you are asked to solve the following IVP:

$$xy' - 2y = -x^2, \qquad y(1) = 0$$

Find the integrating factor that you would need in order to solve this differential equation. NOTE: you do not have to solve the IVP, just find the integrating factor.

A. $\frac{1}{x^2}$ B. $\frac{-1}{x^2}$ C. $e^{(x^2/2)}$ D. $e^{(-x^2/2)}$ E. An integrating factor is not needed to solve this IVP.

11. Consider the function and the statements below:

$$f(x) = \begin{cases} -x^2 + 1 & \text{if } x < 0 \\ \cos(x) & \text{if } x \geq 0 \end{cases}$$

P: f is continuous at $x = 0$

Q: f is differentiable at $x = 0$

R: f' is differentiable at $x = 0$

Which of the above statements are true?

A. P only B. Q only C. P and Q only D. Q and R only E. P, Q, and R

12. Evaluate $\lim_{x \to 3} \dfrac{\sqrt{x+6} - 3}{x^3 - 3x^2}$

A. 0 B. $-\frac{5}{54}$ C. $\frac{1}{54}$ D. 1 E. $\frac{1}{3}$

13. Find $\frac{dy}{dx}$ for the function $y = a^{\sin^2(bx)}$

A. $2\sin(bx)a^{\sin^2(bx)-1}$ B. $2b\sin(bx)a^{\sin^2(bx)-1}$ C. $2b\ln(a)\sin(bx)a^{\sin^2(bx)-1}$

D. $b\ln(a)\sin(2bx)a^{\sin^2(bx)}$ E. $2b\ln(a)\sin(bx)a^{\sin^2(bx)}$

14. Evaluate $\int_1^2 x\sqrt{x-1}\,dx$

A. $\frac{16}{15}$ B. $\frac{2}{3}$ C. 1 D. 0 E. $\frac{6}{5}$

15. Find the value(s) of x in the interval $[0,2]$ for which the tangent line to the curve $f(x) = 2x^2 - 3x + 1$ has the same slope as the secant line passing through the endpoints of this same curve on this same interval.

A. $\frac{1}{3}$ B. $\frac{2}{3}$ and $\frac{4}{3}$ C. $\frac{2}{3}$ D. 1 E. None of these

16. Which of the following integrals will yield the volume generated by rotating the region bounded by $y = x^2$ and $y = 2 - x^2$ about the line $x = 1$?

A. $2\pi\int_{-1}^{1}(1-x)(2-2x^2)\,dx$ B. $2\pi\int_0^1(1-x^2)^2\,dx$ C. $2\pi\int_0^1\left[(2-x^2)^2 - x^2\right]dx$

D. $2\pi\int_{-1}^{1}x(2-x^2)\,dx$ E. $2\pi\int_0^1(1-2x^2)^2\,dx$

17. Evaluate the integral $\int_0^1 \frac{dx}{(x^2+1)^2}$

A. $\frac{\pi+2}{4}$ B. $\ln(2)+\frac{\pi}{4}$ C. $\frac{\pi+2}{8}$ D. $\frac{\sqrt{2}+1}{4}$ E. $\frac{\pi+4}{8}$

18. If the nth partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = 3 - \frac{n}{2^n}$, then what is the value of a_2?

A. $-\frac{1}{2}$ B. $\frac{5}{2}$ C. 0 D. $\frac{1}{4}$ E. 1

19. Using integration by parts, evaluate the following integral if $b > 0$

$$\int x^b \ln(x)\, dx$$

A. $bx^{b-2} + C$ B. $\frac{x^{b+2}}{b+1}(\ln(x) - 1) + C$ C. $x^{b-1}(b\ln(x) + 1) + C$

D. $x^{b-1}\left(1 - \frac{b}{b-1}\right) + C$ E. $\frac{x^{b+1}}{b+1}\ln(x) + \frac{x^{b+1}}{(b+1)^2} + C$

20. Find the area enclosed by one loop of the curve $r = \sin(4\theta)$

A. $\frac{\pi}{16}$ B. $\frac{\pi+\sqrt{3}}{8}$ C. $\frac{2\pi+\sqrt{3}}{16}$ D. $\frac{2\pi+\sqrt{2}}{16}$ E. $\frac{\pi}{8}$

21. Find the sum of the infinite geometric series $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{3(10^{n+1})}$

A. $\frac{1}{5}$ B. $\frac{1}{3}$ C. $\frac{2}{15}$ D. $\frac{4}{15}$ E. $\frac{2}{3}$

22. Evaluate the following indefinite integral

$$\int (\sin x - \cos x)^2\, dx$$

A. $x + \frac{1}{2}\cos(2x) + C$ B. $x + \cos^2(x) + C$ C. $x - \sin^2(x) + C$

D. All of the above are correct E. None of the above are correct

23. Evaluate the following integral, if it exists

$$\int_0^{\infty} xe^{-x^2}\, dx$$

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. The integral does not converge

24. An alien spaceship blasts off from its home planet of Tematyx. Its velocity (in ft/s) t seconds after launch is given by $v(t) = e^t + 0.1t^2 - t - 1$. What is the spaceship's average velocity (in ft/s) over the first 7 seconds of flight? Round to the nearest tenth of a ft/s.

A. 153.6 B. 156.2 C. 214.0 D. 244.6 E. 1,093.5

25. Over what interval(s) is the function $g(x) = \int_2^x (2t+1)\, dt$ increasing?

A. $\left(-\frac{1}{2}, \infty\right)$ B. $(1, \infty)$ C. $(-\infty, \infty)$ D. $(-\infty, -1) \cup (0, \infty)$ E. $(-\infty, -3) \cup (2, \infty)$