TMATYC - Calculus B Test - 2015

1. Evaluate the following integral

$$\int e^{2015x}\,dx$$

A. 2015 B. $e^{2015x}+C$ C. $2015x+C$ D. $\frac{1}{2015}x+C$ E. $\frac{1}{2015}e^{2015x}+C$

2. Evaluate $f'(0)$ if

$$f(x) = \frac{x-3x^2+4x^4+8x^5}{6+x^2-7x^3+11x^4+7x^8}$$

A. 0 B. $\frac{1}{6}$ C. $-\frac{1}{42}$ D. $-\frac{6}{7}$ E. $\frac{6}{7}$

3. Evaluate the following limit, if it exists

$$\lim_{x\to 7}\left(\frac{x}{x-7}\int_7^x \frac{\sin t}{t}\,dt\right)$$

A. $\sin 7$ B. $\cos 7$ C. $\frac{\pi}{7}$ D. 1 E. The limit does not exist

4. Find the absolute value of the Jacobian of the transform T if $x = r\cos\theta$ and $y = r\sin\theta$.

A. 1 B. r^2 C. r D. $2r$ E. θ

5. Evaluate $\iint_D (2x+y)\,dA$, where D is the region bounded by the parabolas $y = x^2+1$ and $y = 2x^2$.

A. $-\frac{178}{15}$ B. $-\frac{16}{15}$ C. $\frac{31}{30}$ D. $\frac{16}{15}$ E. $\frac{32}{15}$

6. Suppose a curve is given by the parametric equations

$$x = 2+\ln t \qquad y = 2t^2+1$$

Find the equation of the tangent line to this curve at the point (2,3).

A. $y = 3$ B. $y = 2x-1$ C. $y = 4x-5$ D. $y = x+1$ E. $y = \frac{1}{4}x+\frac{5}{2}$

7. Find the exact length of the curve below over the given interval

$$y = \ln(\sec x) \qquad 0 \le x \le \frac{\pi}{3}$$

A. $\ln\left(2-\sqrt{3}\right)$ B. $\ln\left(2+\sqrt{3}\right)$ C. $\ln\left(1-\sqrt{2}\right)$ D. $\ln\left(1+\sqrt{2}\right)$ E. $\ln\left(\sqrt{3}-2\right)$

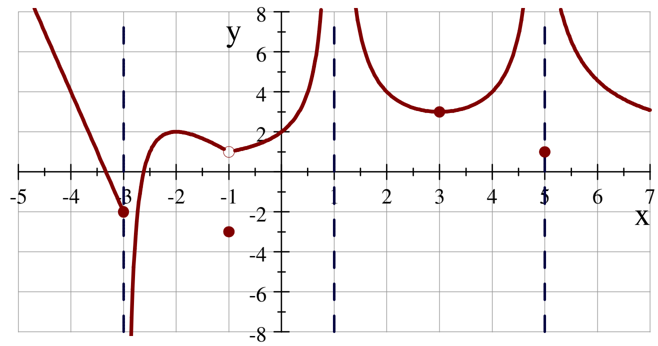
8. Suppose a particle is in back-and-forth linear motion, where motion going left-to-right is considered "going in the positive direction", and its position is given by the function $p(t) = t+2\sin t\cos t$. At time $t = \frac{\pi}{6}$, the particle is

A. moving left-to-right and decelerating B. moving left-to-right and accelerating
C. moving right-to-left and decelerating D. moving right-to-left and accelerating
E. the particle is at rest

9. Find a curve in the xy-plane that passes through the point (2,5) and whose slope at the point (x, y) is given by $2 - \frac{y}{x}$.

A. $y = x + 3$ B. $y = 2x + 1$ C. $y = \frac{14}{x} - x$ D. $y = \frac{6}{x} + x$ E. $y = x^2 + \ln x$

10. List the x-values in the domain of f (pictured below) where local minimums occur. The dashed lines indicate vertical asymptotes for the graph of f.



A. $\{-3, -1, 3, 5\}$ B. $\{-3, 3, 5\}$ C. $\{-1, 3, 5\}$ D. $\{3\}$ E. $\{-1, 3\}$

11. What is the area of the parallelogram spanned by the vectors $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{r} = 7\mathbf{i} - 4\mathbf{j}$ in square units?

A. 26 B. $13\sqrt{5}$ C. 13 D. $\sqrt{15}$ E. 29

12. Suppose $f(e) = \frac{1}{e}$, $g(e) = \frac{5}{e}$, $f'(e) = -\frac{3}{e}$, and $g'(e) = -\frac{2}{e}$. Now let

$$h(x) = \frac{x \ln(x) f(x)}{g(x)}$$

and find the value of $h'(e)$.

A. $\frac{-17e + 10}{5}$ B. $\frac{-13e + 10}{25}$ C. $\frac{-13}{5}$ D. $\frac{10}{e^2}$ E. $\frac{-1}{5e^2}$

13. If p is a continuous function such that $\int_0^{10} p\left(\frac{x}{5}\right) dx = 35$, then $\int_2^3 p(2t - 4)\, dt =$

A. 14 B. 7 C. 3.5 D. 1.75 E. 0

14. Determine if the following series converges of diverges

$$\sum_{n=1}^{\infty} n e^{-\sqrt{n}}$$

A. Converges via the ratio test B. Converges via the integral test

C. Converges via the comparison test D. Converges via the root test

E. This series diverges

15. Find the value of the following integral:

$$\int_{-1}^{1}\left(\frac{x^2-1}{x^4-1}\right)dx$$

A. 2π B. 0 C. $-\frac{23}{30}$ D. $\frac{\pi}{2}$ E. This integral diverges

16. Evaluate the following integral (assume that $a > 0$)

$$\int \frac{dx}{\sqrt{x^2-a^2}}$$

A. $\dfrac{-x}{(x^2-a^2)^{3/2}} + C$ B. $a\tan(\theta) + C$ C. $\ln|\sec(\theta)\tan(\theta)| + C$

D. $\ln\left|x + \sqrt{x^2-a^2}\,\right| + C$ E. $\ln\left(\sqrt{x^2-a^2}\right) + C$

17. Find the 5th degree Taylor Polynomial, centered at $x = 0$, for the function $f(x) = e^x \sin x$:

A. $1 + x + x^2 + x^3 + x^4 + x^5$ B. $x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5$ C. $1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 - \frac{1}{30}x^5$

D. $x + x^2 + \frac{2}{3}x^3 - \frac{4}{5}x^5$ E. $1 + x - \frac{2}{3}x^3 - x^4 - \frac{4}{5}x^5$

18. Find the equation of the tangent plane to curve $z = x^3 - e^y + 1$ at the point $(3, 0, 27)$.

A. $z = 27x - y - 54$ B. $z = 27x + y - 54$ C. $27x - y + 108$

D. $z = -\frac{7}{3}x + 27y + 30$ E. $z = -\frac{7}{3}x - 27y + 30$

19. Find the volume of the solid obtained by rotating the region bounded by $x = (y-3)^2$ and $x = 1$ about the x-axis.

A. 4π B. $\frac{16\pi}{5}$ C. 8π D. $\frac{8\pi}{3}$ E. $\frac{8\pi}{5}$

20. The hydrostatic force, F, in Newtons, against a vertical plane submerged in a liquid is given by

$$F = \delta\int_c^d yL(y)\,dy$$

where $L(y)$ is the horizontal width of plane at depth y, the plane extends from depth $y = c$ to $y = d$, and δ is the weight density of the liquid (in N/m^3).

A 15 meter-long trough is filled with a liquid of weight density 8230 N/m^3. The ends of the trough are equilateral triangles with sides 6 meters long and a vertex at the bottom. Find the hydrostatic force (in Newtons) on one end of the trough. Round your answer to 3 significant digits.

A. 2.65×10^5 B. 3.35×10^5 C. 3.53×10^5 D. 2.44×10^5 E. 2.22×10^5

21. A corporation invests part of its revenue at a rate of P dollars per year in a fund for employee bonuses and benefits. The fund earns r percent interest per year compounded continuously. The rate of growth of the amount A in the fund is represented by the differential equation

$$\frac{dA}{dt} = P + rA$$

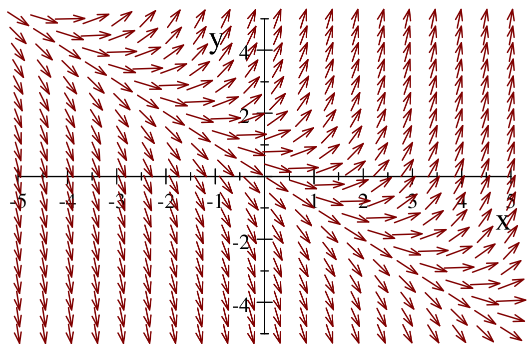
where t is the time (in years). Solve the differential equation for $A(t)$ if the initial amount of the fund is \$0 when they first start at $t = 0$.

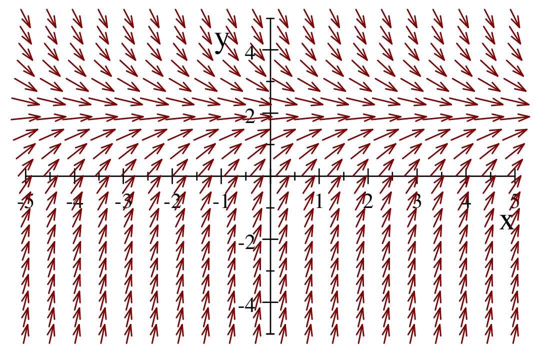
A. $A(t) = \frac{P}{r}e^{rt}$ B. $A(t) = \frac{P}{r}(e^{rt} - 1)$ C. $A(t) = \frac{P}{r}e^{Prt}$

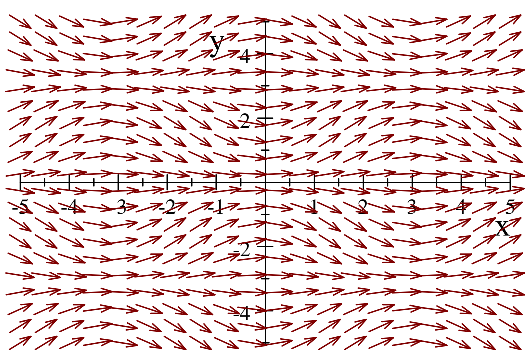
D. $A(t) = \frac{P}{r}(e^{t} - 1)$ E. $A(t) = \frac{P}{r}(e^{rt} + 1)$

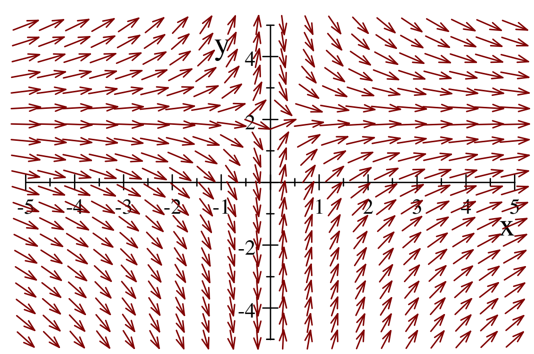
22. Determine the correct slope field for the 1st order differential equation given below

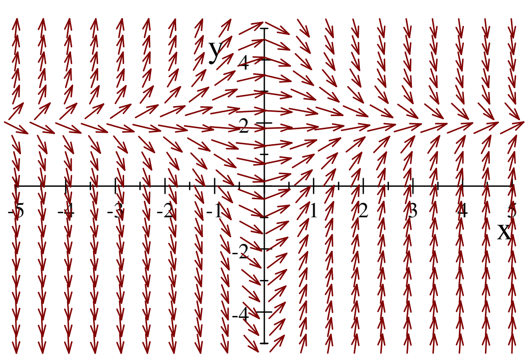
$$\frac{dy}{dx} = 2 - y$$

A. 

B. 

C. 

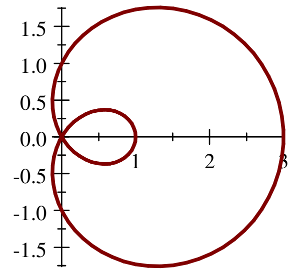
D. 

E. 

23. Define $T : R^2 \to R^2$ by $T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2)$. Find the preimage of $(-1, 11)$

A. $(3,4)$ B. $(-12,21)$ C. $(0,0)$ D. $(2,4)$ E. A preimage does not exist

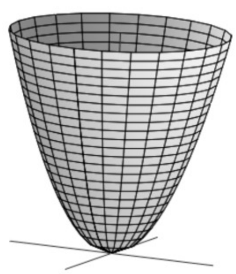
24. The graph of $r(\theta) = 1 + 2\cos\theta$ is pictured below



Find the area inside the inner loop of this polar curve.

A. $\pi - \frac{5\sqrt{3}}{4}$ B. $2\pi - 3\sqrt{3}$ C. $\frac{\pi - 3\sqrt{3}}{3}$ D. $\frac{3\pi}{2} - 2$ E. $\pi - \frac{3\sqrt{3}}{2}$

25. Set up (but don't evaluate) a triple integral in cylindrical coordinates that calculates the volume trapped inside the paraboloid $z = x^2 + y^2$ below the plane $z = 4$. (The volume is shown in the picture.)



A. $\int_0^{2\pi}\int_0^2\int_0^4 r\,dz\,dr\,d\theta$ B. $\int_0^{2\pi}\int_0^2\int_{r^2}^4 r\,dz\,dr\,d\theta$ C. $\int_0^{2\pi}\int_0^2\int_{r^2}^4 dz\,dr\,d\theta$

D. $\int_0^{2\pi}\int_{r^2}^4\int_0^2 r\,dz\,dr\,d\theta$ E. $\int_0^{2\pi}\int_0^2\int_{r^2}^4 r^2\,dz\,dr\,d\theta$