TMATYC - Calculus B Test - 2016

1. Find all the values of a such that the following limit exists: $\lim_{x\to 3} \dfrac{6x^2+(6a+b)x+ab}{2x^2-x-15}$

A. $-\frac{5}{2}$ B. $-5, 3$ C. $-\frac{15}{2}, \frac{5}{2}$ D. -3 E. there are no such values of a

2. If $f(x) = x^2 + \sin x + 2$, then $\lim_{x\to\pi} \dfrac{f(x)-f(\pi)}{x-\pi} =$

A. π^2+2 B. $2\pi-1$ C. $2\pi+1$ D. 0 E. the limit does not exist

3. The ceiling function, denoted $\lceil x \rceil$, is defined as the smallest integer greater than or equal to x. If $g(x) = \lceil x \rceil$, then find $g'(5)$.

A. -1 B. 0 C. 5 D. 6 E. g is not differentiable at $x = 5$

4. If $w = \sqrt{\ln p}$, then $\dfrac{dw}{dp} =$

A. $\dfrac{1}{2p\sqrt{\ln p}}$ B. $\dfrac{1}{2\sqrt{\ln p}}$ C. $\sqrt{\dfrac{1}{p}}$ D. $\dfrac{1}{2p}$ E. $\sqrt{\dfrac{p}{p+1}}$

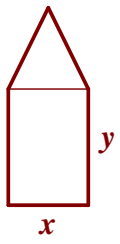
5. Which of the following points on the curve $y^3 - xy + x^3 = 1$ is where the tangent line has slope -1?

A. $(1,-1)$ B. $(1,1)$ C. $(-1,1)$ D. $(1,0)$ E. $(0,1)$

6. Let $p(x) = -x^2 + a$ and $q(x) = (x+1)^2 - b$ where a and b are real numbers. If the line tangent to $p(x)$ at $x = 1$ and the line tangent to $q(x)$ at $x = 1$ intersect at the point $(2,3)$, then $a + b =$

A. 2 B. 5 C. 10 D. 11 E. 13

7. A window is to be constructed in the shape of a rectangle surmounted by an equilateral triangle with base equal to the width of the rectangle as shown below. Note that x is the width and y is the height of the rectangular portion of the window.



If the perimeter of the window is 10 feet, what is the value of x (to the nearest tenth of a foot) that will maximize the surface area of the window?

A. 1.7 B. 2.3 C. 3.3 D. 4.7 E. 5.9

8. Partition the interval $[-1,2]$ into n equally spaced subintervals of width $\Delta x = \frac{3}{n}$ and let c_i be an arbitrary value in the ith subinterval. If $f(x) = 3x^2 - 2x + 1$, the find the value of

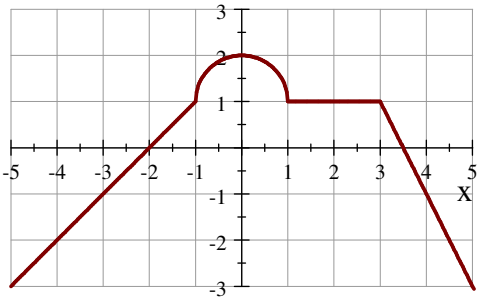
$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

A. 0 B. 1 C. 9 D. ∞ E. does not exist

9. If $g(x) = \int_1^{x^2} (\sin(t^3))\,dt$, then $g'(x) =$

A. $\sin(x^3)$ B. $\sin(x^6)$ C. $2x\sin(x^6)$ D. $\cos(x^6)$ E. $3x^2\cos(x^3)$

10. Given the graph of the function h below, find the value of $\int_{-4}^{4} h(x)\,dx$.



A. $\frac{1}{2}(\pi + 5)$ B. $\frac{1}{2}(4\pi + 5)$ C. $\frac{1}{2}(\pi + 8)$ D. $\frac{1}{2}(\pi + 14)$ E. $\frac{1}{2}(2\pi + 19)$

11. $\int \frac{\ln(\ln t)}{t \ln t}\,dt =$

A. $\frac{1}{2}(\ln t)^2 + C$ B. $\frac{1}{2}[\ln(\ln t)]^2 + C$ C. $\ln(\frac{1}{t}) + C$ D. $\frac{1}{\ln t} + C$ E. $\frac{1}{\ln(\ln t)} + C$

12. A population of insects increases at a rate of $0.3t^2 + 20t + 100$ insects per day (t in days). Find the insect population after 10 days, assuming that there are 100 insects at $t = 0$.

A. 330 B. 430 C. 2100 D. 2200 E. 2315

13. $\int x^2\sqrt{x-1}\,dx =$

A. $\frac{2}{7}x^{7/2} - \frac{1}{3}x^3 + C$ B. $\frac{2x^3(x-1)^{3/2}}{9} + C$ C. $2x\sqrt{x-1} + \frac{x^2}{2\sqrt{x-1}} + C$

D. $\frac{2}{7}(x-1)^{7/2} + \frac{2}{3}(x-1)^{3/2} + C$ E. $\frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

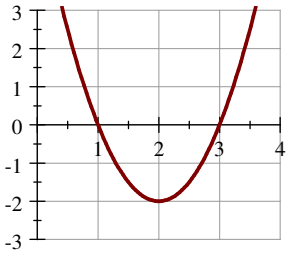
14. The average value of the function f on the interval $[a, b]$ is the quantity

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

If the average value of $f(x)$ on the interval $[0, t]$ is equal to t for all real t, then $f(x) =$

A. $\frac{1}{2}x^2$ B. $\frac{1}{2}x$ C. $2x$ D. e^x E. $\ln x$

15. The graph of f is shown below.



If $g(x) = \int_0^x f(t)\,dt$, then g has a relative minimum at $x =$

A. 0 B. 1 C. 2 D. 3 E. 4

16. A solid is generated by revolving about the x-axis the region bounded by the graph of the positive continuous function $y = f(x)$, the x-axis, and the lines $x = 0$ and $x = b$. Which of the following integrals will yield the solid's volume?

A. $\pi \int_0^b [f(x)]^2\,dx$ B. $\frac{1}{2}\int_0^b [f(x)]^2\,dx$ C. $\int_0^b f(x)\,dx$ D. $\int_0^b 2\pi f(x)\,dx$ E. $\frac{1}{b}\int_0^b f(x)\,dx$

17. If $f(x)$ is an odd function and $g(x)$ is an even function, which of the following might be FALSE?

A. $\int_{-a}^a f(x)\,dx = 0$ B. $\int_{-a}^a [f(x)]^2\,dx = 0$ C. $\int_{-a}^a (fg)(x)\,dx = 0$

D. $\int_{-a}^a g(x)\,dx = 2\int_0^a g(x)\,dx$ E. $\int_{-a}^a [g(x)]^2\,dx = 2\int_0^a [g(x)]^2\,dx$

18. Solve the initial value problem: $\frac{dy}{dx} = e^{-x-y-2}, \quad y(0) = -2$

A. $y = 2(e^{-2} - e^{-x-2} - 1)$ B. $y = -2 + \ln(e^{-2} - e^{-x-2} + 1)$ C. $y = -2\ln\left(e^{x+1} - \frac{1}{2}x^2\right)$

D. $y = -2 + \ln(2 - e^{-x})$ E. $y = -x - 2 - \ln(2 - e^x)$

19. $\int x^5 \ln x\,dx =$

A. $\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C$ B. $\frac{1}{2}x^5(\ln x)^2 - \frac{1}{6}x^6 + C$ C. $5x^4 \ln x + x^4 + C$

D. $\frac{1}{6}x^5 + C$ E. $-\frac{1}{4}x^4 + C$

20. $\sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}} =$

A. $\frac{8}{3}$ B. $\sqrt{7}$ C. 3 D. 6 E. ∞

21. The series $\sum_{n=1}^{\infty} \frac{(-7)^{n+1}}{3^{2n}}$

A. diverges B. neither converges nor diverges C. converges with sum $-\frac{49}{9}$

D. converges with sum $\frac{49}{2}$ E. converges with sum $\frac{49}{16}$

22. Find the equation of the line tangent to the curve whose parametric equations are

$$x = \frac{1}{t+1}, \quad y = \frac{t}{t-1}$$

at $t = 2$.

A. $y = -\frac{1}{9}x + \frac{55}{27}$ B. $y = -x + \frac{7}{3}$ C. $y = x + \frac{5}{3}$ D. $y = 6x$ E. $y = 9x - 1$

23. Find the velocity, $\vec{v}(t)$, and the speed, $|\vec{v}(t)|$, of a particle moving along a smooth curve in space whose position vector is given by

$$\vec{r}(t) = 3\sin t\,\mathbf{i} - 3\cos t\,\mathbf{j} + 4t\,\mathbf{k}$$

A. $\vec{v}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4\,\mathbf{k}$; $|\vec{v}(t)| = \sqrt{10}$ B. $\vec{v}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4\,\mathbf{k}$; $|\vec{v}(t)| = 5$

C. $\vec{v}(t) = 3\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 4\,\mathbf{k}$; $|\vec{v}(t)| = \sqrt{7}$ D. $\vec{v}(t) = -3\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j} + 4\,\mathbf{k}$; $|\vec{v}(t)| = 5$

E. $\vec{v}(t) = -3\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j} + 4\,\mathbf{k}$; $|\vec{v}(t)| = \sqrt{7}$

24. Find the function $w = f(x,y)$ if $\frac{\partial^2 w}{\partial^2 x} = 6x + e^x\cos y$, $\frac{\partial^2 w}{\partial^2 y} = -2 - e^x\cos y$, $\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x} = -e^x\sin y$, and such that $f(0,0) = 3$

A. $w = x^3 - y^2 - e^x\cos y + 4$ B. $w = x^3 - y^2 - e^x\sin y + 3$ C. $w = x^3 - y^2 + e^x\cos y + 2$

D. $w = 3x^2 - 2y + e^x\cos y - e^x\sin y + 2$ E. $w = x^3 + e^x\cos y + 2$

25. The radioactive isotope thorium-234 disintegrates at a rate proportional to the amount present. If 100 mg of this material is reduced to 80 mg in one week, what is the time interval (to the nearest tenth of a week) that must elapse for the mass to decay to one-half of its original value?

A. 2.2 B. 2.5 C. 2.9 D. 3.1 E. 3.6