TMATYC - Calculus B Test - 2017

1. The derivative with respect to x of the function $y = 2\tan^{-1}\sqrt{\dfrac{x-\beta}{\alpha-x}}$, where α and β are positive constants, is

A. $\dfrac{\alpha-\beta}{\sqrt{(x-\beta)(\alpha-x)^3}}\sec^2\sqrt{\dfrac{x-\beta}{\alpha-x}}$ B. $\sec^2\sqrt{\dfrac{x-\beta}{\alpha-x}}$ C. $\dfrac{\alpha-x}{\alpha-\beta}$

D. $\sqrt{(x-\beta)(\alpha-x)}$ E. $\dfrac{1}{\sqrt{(x-\beta)(\alpha-x)}}$

2. $\lim\limits_{x\to\infty}\left[(x^2+ax+b)^{1/2}-x\right] =$

A. 0 B. $a+b$ C. $\sqrt{b}$ D. $\dfrac{a}{2}$ E. ∞

3. What is the domain of $g'(x)$ if $g(x) = x^{5/2}(x-2)^{2/3}$?

A. $(-\infty,\infty)$ B. $[0,\infty)$ C. $[2,\infty)$ D. $(-\infty,2)\cup(2,\infty)$ E. $[0,2)\cup(2,\infty)$

4. Find the point of intersection of the plane $2x+y+3z = 38$ and the line whose symmetric equations are $\dfrac{x-1}{2} = \dfrac{y-3}{5} = \dfrac{z+1}{3}$.

A. $(1,3,-1)$ B. $(3,8,8)$ C. $(5,13,5)$ D. $(7,18,2)$ E. $(3,8,2)$

5. Find the first three terms of the Taylor Series expansion for the function $f(x) = \tan^{-1}x$ centered about $x = 0$.

A. $x-\frac{1}{3}x^3+\frac{1}{5}x^5$ B. $-x+\frac{1}{3}x^3-\frac{1}{5}x^5$ C. $x+\frac{1}{3}x^3+\frac{1}{5}x^5$ D. $-x-\frac{1}{3}x^3-\frac{1}{5}x^5$ E. $x-2x^3+24x^5$

6. Find the equation of the tangent plane to the surface $4x^2+9y^2+z = 17$ at the point $(-1,1,4)$.

A. $8x+8y+z = 30$ B. $8x+8y-z = -30$ C. $8x-18y-z = -30$

D. $8x-18y-z = 30$ E. $-x+y+4z = 17$

7. All units of a 30-unit apartment building are rented out when the monthly rent is set to \$1000. A survey shows that one unit becomes empty with each \$40 increase in rent. Additionally, each occupied unit costs the landlord \$120 in maintenance per month. What rent will maximize profit?

A. \$800 B. \$1160 C. \$1200 D. \$1320 E. \$1400

8. Evaluate the following determinant

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

A. $(b-a)(c-a)(c-b)$ B. $(a^2-b^2)(c-a)$ C. $a^3b^3c^3$ D. abc E. $c^2(b-a)$

9. Given the curve $\vec{r}(t) = \langle at, b\cos t, b\sin t \rangle$, where a and b are nonzero real constants, find the unit tangent vector $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$

A. $\dfrac{1}{\sqrt{a^2+b^2}}\langle a, -b\sin t, b\cos t \rangle$ B. $\dfrac{1}{\sqrt{a^2+b^2}}\langle b, -a\sin t, a\cos t \rangle$ C. $\dfrac{1}{a+b}\langle a, -b\sin t, b\cos t \rangle$

D. $\langle 1, -1, 1 \rangle$ E. $\langle a, -b\sin t, b\cos t \rangle$

10. Integrate the following using partial fraction decomposition: $\displaystyle\int \frac{5x^2+x+3}{x^3+x}\,dx$

A. $3\ln|x| + \ln(x^2+1) + C$ B. $3\ln|x| + \ln(x^2+1) + \arctan x + C$ C. $\frac{1}{3}\ln|x| + \frac{2}{7}\ln|x+1| + \frac{2}{9}\ln|x-1| + C$

D. $3\ln|x| + \ln|x+1| - \dfrac{1}{x+1} + C$ E. $\dfrac{10x+1}{3x^2+1} + C$

11. Find the volume of the solid formed by rotating $y = x^2$ about the x-axis between $x = 1$ and $x = 2$.

A. $\dfrac{7\pi}{3}$ B. $\dfrac{31\pi}{5}$ C. $\dfrac{14\pi}{3}$ D. $\dfrac{15\pi}{4}$ E. $\dfrac{21\pi}{2}$

12. Solve the following initial value problem.

$$4x\frac{dy}{dx} - \frac{3x^3}{y} = 0, \qquad y(0) = 4$$

A. $y = \frac{1}{4}x^3 + 4$ B. $y = \frac{1}{2}x^{3/2} + 4$ C. $y = \frac{3}{2}\sqrt{x+16}$ D. $y = \sqrt{\frac{1}{2}x^3 + 16}$ E. $y = \left(\frac{3}{4}x^2 + 64\right)^{1/3}$

13. Given that $x = b\sin\left(\frac{a}{b}t\right) + a^2$ and $y = b\cos\left(\frac{a}{b}t\right) + a^2$ for any parameter t, find $\dfrac{dy}{dx}$ in terms of t.

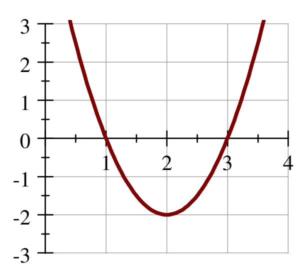
A. $-\cot\left(\frac{a}{b}t\right)$ B. $\tan\left(\frac{a}{b}t\right)$ C. $-\tan\left(\frac{a}{b}t\right)$ D. $-a\sin\left(\frac{a}{b}t\right)$ E. $-b\sin\left(\frac{a}{b}t\right)$

14. Over what interval or intervals is the function $f(x) = 28x^3 - 13x^2 + 2x + 11$ decreasing?

A. $\left(\frac{2}{13}, \frac{2}{11}\right)$ B. $\left(\frac{1}{11}, \frac{2}{13}\right) \cup \left(\frac{2}{11}, \frac{1}{7}\right)$ C. $\left(\frac{1}{7}, \frac{1}{6}\right)$

D. $\left(\frac{2}{13}, \frac{1}{6}\right) \cup \left(\frac{2}{11}, \frac{3}{13}\right)$ E. f is never decreasing

15. The graph of f is shown below.



If $g(x) = \displaystyle\int_0^x f(t)\,dt$, then $g'(2)$ appears to be equal to

A. -2 B. -1 C. 0 D. 1 E. 2

16. If $h(x) = x^{\ln x}$ then $h'(e) =$

A. 1 B. e C. $2e$ D. $\frac{1}{e}$ E. 2

17. $\int \sin x \cos(\cos x)\, dx =$

A. $-\cos x \sin(\sin x) + C$ B. $-\cos^2(\cos x) + C$ C. $\sin x \sin(\cos x) + C$

D. $\sin(\sin x) + C$ E. $-\sin(\cos x) + C$

18. What is the value of the improper integral $\int_1^4 \frac{dx}{(x-2)^2}$

A. $-\frac{1}{2}$ B. $-\frac{3}{2}$ C. $\frac{1}{4}$ D. $\frac{1}{16}$ E. ∞ (the integral diverges)

19. If $P,\ V,\ T,$ and ϕ are related by the equations

$$PV = CT \text{ and } \phi = A \ln P + B \ln V$$

where $A,\ B,$ and C are constants such that $C = B - A,$ then $\frac{\partial T}{\partial P} \cdot \frac{\partial \phi}{\partial V} - \frac{\partial T}{\partial V} \cdot \frac{\partial \phi}{\partial P}$ is equal to

A. 0 B. –1 C. 1 D. –2 E. 2

20. The radius of curvature of the curve $y = \ln(\cos x)$ is

A. $\sec x$ B. $-\sec x$ C. $\tan x$ D. $\sin x$ E. $-\tan^2 x$

21. Find the sum of the infinite geometric series given by $\sum_{n=1}^{\infty} \frac{2^{3n+1}}{3(10^{n+1})}$

A. $\frac{1}{5}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{4}{15}$ E. $\frac{4}{5}$

22. A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light rotates 4 rev/min. How fast is the light beam moving along the shoreline when it is 1 km from P? Give your answer in km/min.

A. 4π B. $\frac{80\pi}{3}$ C. $\frac{27\pi}{4}$ D. 120π E. $\frac{40}{3}$

23. Find the equation of the tangent line at the point $(4at^2, 8at^3)$ of the curve $ay^2 = x^3$ where a is a nonzero constant.

A. $y = 3tx + 4at^3$ B. $y = 3tx - 4at^3$ C. $y = -3tx + 4at^3$ D. $y = -2tx + 16at^3$ E. $y = 2tx$

24. Find the volume of the parallelepiped where $\vec{a} = \langle 1,2,3 \rangle,\ \vec{b} = \langle -1,1,2 \rangle,$ and $\vec{c} = \langle 2,1,4 \rangle$

A. 6 B. 7 C. 8 D. 9 E. 10

25. Find the arc length of the cardioid given by $r = 2(1 - \cos\theta)$ where $0 \le \theta \le 2\pi$.

A. 2 B. 4 C. 4π D. 16 E. 8