TMATYC - Calculus B Test - 2018

1. If $\lim_{x \to 0} \dfrac{\sqrt{ax+b} - \sqrt{3}}{x} = \sqrt{3}$ then $a + b =$

A. 0 B. 3 C. $2\sqrt{3}$ D. 4 E. 9

2. If f is continuous on the interval $[a,b]$ and $f(a) < y < f(b)$, then which of the following **MUST** be true?

A. $f(b)$ must be a maximum for f on the interval $[a,b]$.

B. $f'(x)$ must be positive for all values of x in the interval (a,b).

C. $y = \dfrac{f(a)+f(b)}{2}$

D. There is some number c in the interval (a,b) such that $f(c) = y$.

E. There is some number c in the interval (a,b) such that $f'(c) = 0$.

3. Find the equation of the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3,-4)$.

A. $y = -\frac{3}{4}x - \frac{7}{4}$ B. $y = \frac{3}{4}x - \frac{49}{8}$ C. $y = \frac{3}{4}x - \frac{25}{4}$ D. $y = \frac{4}{3}x - 8$ E. $y = \frac{7}{10}x - \frac{61}{10}$

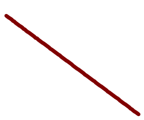
4. If $y = xe^{2x}$, then $\dfrac{d^3y}{dx^3} =$

A. $4e^{2x}(2x+3)$ B. $e^{2x}(x+3)$ C. $e^{2x}(2x+1)$ D. $8x^3e^{2x}$ E. e^{2x}

5. If a and b are constants and $g(x) = \dfrac{ax^2+b}{x^2+1}$, then $g'(1) =$

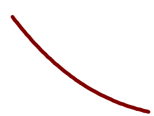
A. a B. $\frac{1}{2}(a-b)$ C. $\frac{1}{2}(a+b)$ D. $\frac{1}{4}(3a-b)$ E. $a+b$

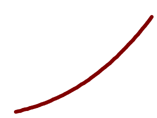
6. If $y' > 0$ and $y'' < 0$ for all x, which of the following is a possible graph for y?

A. 

B. 

C. 

D. 

E. 

7. If the parabola $y = ax^2 + bx + c$ passes through the point $(2,6)$ and is tangent to the line $y = 4x - 11$ at $(3,1)$ then $a + b + c =$

A. 29 B. 30 C. 31 D. 33 E. 35

8. The displacement y, in feet, from equilibrium for an object on the end of a spring is given by

$$y = \frac{1}{5}\cos 3\pi t - \frac{1}{4}\sin 3\pi t$$

where t is the elapsed time in seconds. What is the velocity (in ft/s) of the object at $t = 0.5$ seconds?

A. $\frac{3\pi}{5}$ B. $\frac{3}{5}$ C. $-\frac{1}{4}$ D. $-\frac{\pi}{20}$ E. $-\frac{1}{20}$

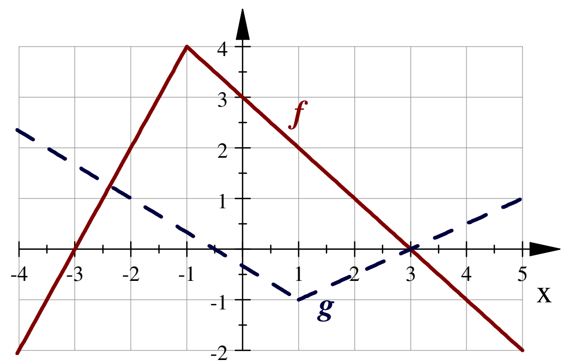
9. Let $T = f(x)$ be a function that gives the temperature x hours after noon. For $0 \le x \le 6,$ the following information is known:

$$f'(x) = 4 \text{ for } 0 \le x \le 3, \qquad \text{and } f''(x) = -2 \text{ for } 3 \le x \le 6$$

When during the 6 hours after noon, is the temperature the highest?

A. $x = 0$ B. $x = 3$ C. $x = 4$ D. $x = 5$ E. $x = 6$

Use the graphs of the functions f (solid line) and g (dashed line) shown below to answer question #s 10 and 11.



10. If $h(x) = f(x)g(x)$ then $h'(-2) =$

A. 2 B. 5 C. -2 D. $\frac{2}{3}$ E. $-\frac{4}{3}$

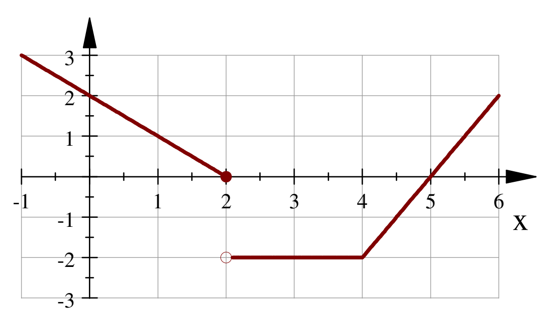
11. If $p(x) = f(x^2)$ then $p'(-1) =$

A. 2 B. 4 C. -2 D. -1 E. 1

12. Two lines are tangent to the graph of $y = 4x - x^2$. If the points of tangency for these two lines occur at $x = 1$ and $x = 3,$ then at what point do these two lines intersect?

A. $(1,3)$ B. $(2,-2)$ C. $(1,9)$ D. $(2,3)$ E. $(2,5)$

13. Estimate $\int_0^5 f(x)\,dx$ given the graph of the function f below.



A. -3 B. -2 C. 1 D. 2 E. 7

14. For the positive constant a, if $\int_a^{2a} \left(\frac{x^2}{a} + 3a \right) dx = 3$, then $4a =$

A. 5 B. 4 C. 3 D. 2 E. 1

15. If b is a constant, then $\int \frac{3bt\,dt}{\sqrt{bt^2 + 1}} =$

A. $3\sqrt{bt^2 + 1} + C$ B. $3b^2 t\sqrt{bt^2 + 1} + C$ C. $\frac{6bt^2}{(bt^2 + 1)^{3/2}} + C$

D. $\frac{3b^2(t^2 + 2)}{2(bt^2 + 1)^{3/2}}$ E. $\frac{3}{4} bt^2 (bt^2 + 1)^{3/2} + C$

16. $\int e^x \sin x \, dx =$

A. $e^x \cos x + C$ B. $-e^x \cos x + C$ C. $\frac{1}{2} e^x (\sin x - \cos x) + C$

D. $\frac{1}{2} e^x \cos x - \sin x + C$ E. $e^x (\sin x + \cos x) + C$

17. $\frac{d}{dx} \left[\int_x^{2x} \sqrt{t^4 + 1} \, dt \right] =$

A. $2\sqrt{x^4 + 1}$ B. $2\sqrt{16x^4 + 1} - \sqrt{x^4 + 1}$ C. $\frac{2x^2(x^4 + 3)}{(x^4 + 1)^{\frac{3}{2}}}$

D. $\frac{16x^3}{\sqrt{16x^4 + 1}} - \frac{2x^3}{\sqrt{x^4 + 1}}$ E. $2\sqrt{2}\, x^{\frac{3}{2}} - \frac{1}{2} x^{\frac{3}{2}}$

18. Find the volume of the solid formed by rotating the region in the first quadrant bounded by the curves $y = x^2$, $y = 0$, and $x = 1$ about the line $x = 2$.

A. $\frac{\pi}{2}$ B. $\frac{\pi}{5}$ C. $\frac{5\pi}{6}$ D. $\frac{17\pi}{15}$ E. $\frac{11\pi}{6}$

19. The velocity of a particle (in feet per second) moving along a line is $v(t) = 5t - 7$. Find the distance traveled by the particle (in feet) during the time interval $0 \le t \le 3$ where t is in seconds.

A. 1.5 B. 2 C. 5 D. 8 E. 11.3

20. Calculate $\int_{-\infty}^{\infty} t f(t)\, dt$ if for some positive constant c we have

$$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ e^{-ct} & \text{if } t \ge 0 \end{cases}$$

A. c^2 B. $\frac{1}{c^2}$ C. $\frac{c}{e^c}$ D. $\frac{1}{ce^c}$ E. ∞

21. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-5)^n x^n}{\sqrt{n}}$

A. $(-\frac{1}{5}, \frac{1}{5})$ B. $(-\frac{1}{5}, \frac{1}{5}]$ C. $(-5, 5]$ D. $(0, 5]$ E. $(-\infty, \infty)$

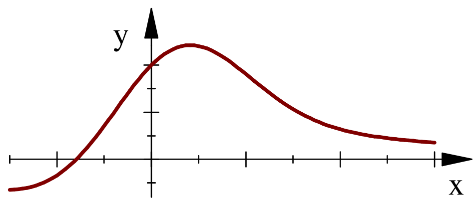
22. If the vector $\vec{u} = \langle -22, 32, -6 \rangle$ is mutually orthogonal to the vectors $\langle a, 3, 5 \rangle$ and $\langle b, 2, -4 \rangle$, then $ab =$

A. -6 B. 0 C. 7 D. 12 E. 14

23. Find the unit tangent vector to the curve defined by the vector function $\vec{r}(t) = (t^3 + 1)\vec{i} + (te^{-t})\vec{j} + \sin(2t)\vec{k}$ at the point where $t = 0$.

A. $\vec{k}$ B. $\frac{1}{\sqrt{2}}\vec{j} + \frac{1}{\sqrt{2}}\vec{k}$ C. $\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$ D. $\frac{1}{\sqrt{5}}\vec{j} + \frac{2}{\sqrt{5}}\vec{k}$ E. $\frac{3}{\sqrt{11}}\vec{i} - \frac{1}{\sqrt{11}}\vec{j} + \frac{1}{\sqrt{11}}\vec{k}$

24. The function y whose graph is shown below is a possible solution to which of the following differential equations?



A. $y' = 1 + xy$ B. $y' = -2xy$ C. $y' = 1 - 2xy$ D. $y' = (x-2)^2$ E. $y' = y - x^2$

25. The length l, width w, and height h of a box change with time. At a certain instant, the dimensions are $l = 1$ ft and $w = h = 2$ ft, and l and w are increasing at a rate of 2 ft/s while h is decreasing at a rate of 3 ft/s. At that instant, find the rate at which the surface area of the box is changing. Express your answer in ft^2/s.

A. -24 B. -16 C. 4 D. 10 E. 16