TMATYC
PRECALCULUS EXAM
Fall 2010

1. If $f(3x-1) = e^{4x-1}$, then $\ln\left(f(5)\right) =$

   A. 4 B. 7 C. 19 D. 55

2. If $\tan\theta = \frac{\sqrt{3}}{3}$ *and* $\sin\theta < 0$, *then* $\cos\theta =$

   A. $\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{3}}{2}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$

3. Find all solutions to the equation $2\sin^2(3x+4) = 1$.

   A. $\frac{(2n+1)\pi - 16}{12}$ B. $\frac{(4n+1)\pi - 16}{12}$

   C. $\frac{(8n+1)\pi - 16}{12}$ D. $\frac{(4n+1)\pi}{12}$

4. If the zeros of the polynomial function $P(x) = x^2 + bx + c$ are $\ln 2$ and π, then $b =$

   A. $\pi \ln 2$ B. $2\pi \ln 2$ C. $\ln 2 + \pi$ D. $-(\ln 2 + \pi)$

5. The path of a projectile fired at an inclination of θ degrees to the horizontal with an initial velocity of V_o is a parabola. The range of the projectile – that is the horizontal distance that the projectile travels – is given by:

$$R = \frac{V_0^{\,2}\sin(2\theta)}{g}$$

   where g is the acceleration due to gravity. Suppose that the projectile is fired with an initial velocity of 400 ft/sec and $g = 32\ ft/sec^2$. What angle θ *for* $0 \le \theta \le 90°$ should you select for the range, R to be 2500 ft.?

   A. $75°$ B. $15°$ C. $15°$ *or* $75°$ D. $-15°$ *or* $75°$

6. If $h(x) = 2x - 8$, *find* $h^{-1}(6)$.

   A. -4 B. ¼ C. 7 D. 11

7. The ideal gas law states that the volume V that a gas occupies is directly proportional to the product of the number m of the moles of gas and the temperature T and is inversely proportional to the pressure P (in atmospheres). Let K be the constant of proportionality, then,

$$V = \frac{KmT}{P}$$

What is the effect on the volume V if the number of moles is doubled and both the temperature and the pressure are reduced by a factor of ½.

A. original volume is doubled

B. no effect

C. ½ original volume

D. volume increases by a factor of 4

8. The radiator of a car needs to contain 40 liters of a 40% antifreeze solution. The radiator now contains 40 liters of a 20% antifreeze solution. How many liters of this solution must be drained and replaced with 100% antifreeze to get the desired strength?

A. 16 liters B. 13.3 liters C. 10 liters D. 20 liters

9. If you have an n-sided polygon inscribed in a circle of r. express the perimeter P in terms of n and r.

A. $nr\left(\frac{360}{n}\right)^{\circ}$ B. $2\pi r\left(\frac{360}{n}\right)^{\circ}$ C. $nr\left(\frac{180}{n}\right)^{\circ}$ D. $2\,nr\,sin\left(\frac{180}{n}\right)^{\circ}$

10. Find the sum of the x and y intercepts of the line with slope $-\frac{1}{3}$ which is the hypotenuse of a right triangular region in Quadrant I with legs the x and y axes and area $\frac{392}{3}$.

A. $\frac{28}{3}$ B. $\frac{56}{3}$ C. 28 D. $\frac{112}{3}$

11. Circle O has equation $x^2 + y^2 = 16$. If P is (1, 0) and Q is (-1, 0) and R is any point on circle O, what is the largest possible value of $PR \mid QR$?

A. 8 B. $2\sqrt{17}$ C. $6\sqrt{2}$ D. $\frac{17}{2}$

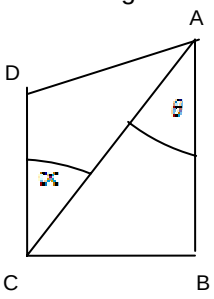
12. A horse is racing around a circular track that has a radius of 840 feet. A camera sits at the center of the circle that forms the track. What angle (to the nearest tenth of a degree) must the camera turn through in order to film the horse running 500 ft. around the track?

A. 36.5^{o} B. 0.8^{o} C. 34.1^{o} D. 53.5^{o}

13. Two triangles ΔABC and ΔADC are shown below. Angles $< ABC$ and $< ADC$ are right angles of $\overline{AB} = 20$, $\overline{AD} = 8$ and $\theta = 20^o$, then find the angle α to the nearest degree (see figure I below).

A. 23^o B. 22^o C. 24^o D. 20^o

Figure I



14. Find an equation of the line that is perpendicular to $-2x - y = 8$ and contains (0, 4).

A. $y = \frac{1}{2}x + 4$ B. $y = -2x - 4$ C. $y = -\frac{1}{2}x - 4$ D. $y = 2x + \frac{1}{4}$

15. The equatorial radius of Earth is approximately 3963.3 miles. Find the linear speed of a point on the equator as a result of Earth's rotation (approximately).

A. 2200 mi/hr B. 1640 mi/hr C. 1040 mi/hr D. 1260 mi/hr

16. Find an equation using the cotangent function that has the same graph as $y = \tan x$.

A. $y = -\cot\left(x + \frac{\pi}{2}\right)$ B. $y = \cot\left(x - \frac{\pi}{2}\right)$

C. $y = \cot(x) + \frac{\pi}{2}$ D. $y = -\cot x - \frac{\pi}{2}$

17. Find the horizontal asymptote for the graph of $f(x) = \dfrac{3x-1}{x^2 - x - 6}$, if it exists.

A. Does not exist B. $y = -3$ C. $y = \frac{1}{3}$ D. $y = 0$

18. It has been estimated that 1000 curies of a radioactive substance introduced at a point on the surface of the open sea would spread over an area of 40,000 km^2 in 40 days. Assuming that the area covered by the radioactive substance is a linear function of time t and it's always circular in shape, express the radius r of the contamination of a function t.

A. $r(t) = \sqrt{\frac{1000t}{\pi}}$ B. $r(t) = \frac{1000}{2\pi} t$

C. $r(t) = t\sqrt{\frac{1000}{\pi}}$ D. $r(t) = \frac{t\sqrt{1000}}{\pi}$

19. What is $arc\csc\frac{5}{4} + arc\sec\frac{5}{4} + arc\cot\left(\frac{5}{4}\right) + arccot\left(\frac{4}{5}\right)$

A. π B. 2π C. $\frac{\pi}{4}$ D. $\frac{\pi}{2}$

20. Suppose $f(x) = ax + b$, $g(x) = bx + a$ (a and b integers). If $f(1) = 8$ and $|f(g(50)) - g(f(50))| = 28$ find the product of a and b.

A. 16 B. 7 C. 12 D. 182

21. As $x \to \left(\frac{\pi}{2}\right)^-$, $\sec x \to$?

A. $-\infty$ B. π C. $-\pi$ D. ∞

22. Given $y = 3\cos\left(x + \frac{\pi}{6}\right)$, what is the amplitude, period and phase shift?

A. $3, 2\pi, -\frac{\pi}{6}$ B. $3, -\pi, \frac{\pi}{6}$ C. $3, \pi, \frac{\pi}{6}$ D. $3, \frac{\pi}{2}, \frac{\pi}{6}$

23. What is the horizontal asymptote of $f(x) = \left(1 + \frac{2}{x}\right)^x$?

A. $y = e$ B. $y = \frac{e}{2}$ C. $y = e^2$ D. $y = \frac{e^2}{2}$

24. Simplify the expression: $\frac{2 - \tan\theta}{2\csc\theta - \sec\theta}$.

A. $\sin\theta$ B. $\cos\theta$ C. $\tan\theta$ D. $-\cos\theta$

25. Find the zeros of $f(x) = -x^2e^{-x} + 2xe^{-x}$.

A. 0, 1 B. 0, 2 C. -1, 2 D. 0, -1