TMATYC - Precalculus Test - 2017

1. Which of the following pairs of complex numbers has both a product and sum that are real numbers?

A. $3 + 4i$ and $6 - 4i$ B. $3 + 4i$ and $1 + 4i$ C. $4i$ and $5i$ D. $3 + 2i$ and $3 - 2i$ E. $3 + 4i$ and $4 + 3i$

2. Given the polynomial function $f(x) = ax^4 + 4x^2 + 1$, which of the following is true if $a > 0$?

A. The graph of f is symmetric about the y-axis B. The graph of f is symmetric about the x-axis

C. The graph of f has a vertical asymptote D. The graph of f has a horizontal asymptote

E. The function f has exactly 4 real zeros (x-intercepts)

3. What is the only value listed below that is **not** the exact value of the sine, cosine, or tangent of any angle that is a multiple of $\pi/6$ or $\pi/4$.

A. $-\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{2}}{3}$ C. 1 D. $\frac{\sqrt{2}}{2}$ E. $-\frac{1}{2}$

4. Find the exact sum of the series $\sum_{k=0}^{\infty} \left(\frac{7}{8}\right)^k$

A. 8 B. $\frac{7}{8}$ C. $\frac{8}{7}$ D. $\frac{49}{64}$ E. ∞ (The series does not converge.)

5. Find the radius of the circle having equation $4x^2 + 4y^2 - 8x + 2y - 1 = 0$

A. 1 B. $\frac{\sqrt{21}}{4}$ C. $\frac{1}{2}$ D. $3\sqrt{2}$ E. There is no circle. The radius would be negative.

6. Solve the following inequality. Write the solution using interval notation.

$$\frac{3x^2(2x-1)}{(x+1)^2(x-3)} \leq 0$$

A. $(-\infty, 0) \cup [\frac{1}{2}, \infty)$ B. $(-\infty, 0) \cup [3, \infty)$ C. $\{0\} \cup [\frac{1}{2}, 3)$ D. $(-\infty, -1) \cup \left[0, \frac{1}{2}\right] \cup [3, \infty)$ E. $[0, 3)$

7. Assuming a, b, c, and d are positive real numbers, use Descartes' Rule of Signs to find the maximum possible number of negative solutions to the following equation.

$$ax^3 - bx^2 - cx + d = 0$$

A. 0 B. 1 C. 2 D. 3 E. Descartes' Rule of Signs does not apply

8. Write the following expression in terms of $\log_3 x$, $\log_3 y$, and $\log_3 z$ only.

$$\log_3\left(\frac{\sqrt[3]{x}}{y^2 z}\right)$$

A. $3\log_3 x + \log_3 y + \log_3 z$ B. $3\log_3 x + 2\log_3 y + \log_3 z$ C. $\frac{1}{3}\log_3 x - 2\log_3 y + \log_3 z$

D. $\frac{1}{3}\log_3 x - 2\log_3 y - \log_3 z$ E. $\log_3 x + 2\log_3 y - \log_3 z$

9. Use the Sine Sum Formula to rewrite the following as an algebraic expression.

$$\sin\left(\arctan x + \arccos\frac{1}{3}\right)$$

A. $\frac{1}{3}x$ B. $\dfrac{1}{\sqrt{1+x^2}} + \dfrac{2\sqrt{2}}{3}$ C. $\dfrac{x+2\sqrt{2}}{3\sqrt{1+x^2}}$ D. $\dfrac{2\sqrt{2}}{9} - \dfrac{x}{1+x^2}$ E. $\dfrac{x-1}{3\sqrt{1+x^2}}$

10. Find the dot product, $\mathbf{v} \cdot \mathbf{w}$, of the given vectors.

$$\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} \qquad \mathbf{w} = \mathbf{i} - 5\mathbf{j}$$

A. 1 B. $2\mathbf{i} - 15\mathbf{j}$ C. $3\mathbf{i} - 2\mathbf{j}$ D. -13 E. -20

11. Find the phase shift of the trigonometric function $g(x) = -3\cos(4x + \pi)$

A. $\frac{\pi}{3}$ B. 3 C. $\frac{\pi}{2}$ D. -3 E. $-\frac{\pi}{4}$

12. Find the inverse function, $f^{-1}(x)$, for $f(x) = 2x^3 - 1$.

A. $f^{-1}(x) = 2x^3 + 1$ B. $f^{-1}(x) = \sqrt[3]{\dfrac{x+1}{2}}$ C. $f^{-1}(x) = 2\sqrt[3]{x} + 1$

D. $f^{-1}(x) = \dfrac{x^3 - 1}{2}$ E. f is not invertible

13. Find the equation of the horizontal or oblique (slant) asymptote of the following rational function.

$$R(x) = \frac{4x^2 + x - 3}{2x + 1}$$

A. Horizontal asymptote of $y = 2$ B. Oblique (slant) asymptote of $y = 2x$

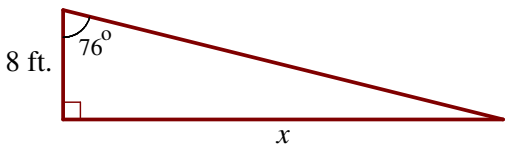
C. Horizontal asymptote of $y = -3$ D. Horizontal asymptote of $y = 0$

E. Oblique (slant) asymptote of $y = 2x - \frac{1}{2}$

14. Let $(x, y) = (1, -4)$ be a point in rectangular coordinates. Convert that point to polar coordinates in the form (r, θ), with θ in the interval $[0, 2\pi)$.

A. $\left(\sqrt{17}, 2\pi - \arctan(4)\right)$ B. $\left(\sqrt{17}, 4\right)$ C. $\left(\sqrt{17}, \frac{\pi}{4}\right)$ D. $(4, -1)$ E. $(17, 4)$

15. Find the value of x, to the nearest foot.



A. 8 B. 12 C. 24 D. 32 E. 608

16. Use the Rational Root Theorem (Rational Zeros Theorem) to create a list of **all possible** rational solutions to the following equation.

$$3x^5 - 4x^4 - 8x^2 + 2x + 6 = 0$$

A. $\pm 2, \pm 3, \pm 4, \pm 6, \pm 8$ B. $\pm 1, \pm 2, \pm 3, \pm 6$ C. $\pm 1, \pm 2, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

D. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{3}, \pm \frac{2}{3}$ E. $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

17. Find all solutions to the following equation in the interval $[0, 2\pi)$.

$$2\sin^2 x + \sin x - 1 = 0$$

A. $\left\{0, \frac{\pi}{2}, \pi\right\}$ B. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ C. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ D. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{\pi}{2}\right\}$ E. No solution

18. Find the solution (x, y, z) to the following system of linear equations.

$$x + 3y + z = 2$$
$$2x - y + 3z = 5$$
$$8x + 3y + 11z = 19$$

A. $(2, -1, 3)$ B. $(-\frac{3}{7}, \frac{6}{7}, -\frac{1}{7})$ C. $(\frac{17}{7}, -\frac{1}{7}, 0)$ D. The system is inconsistent. There is no solution.

E. There are infinitely many solutions of the form $(-\frac{10}{7}t + \frac{17}{7}, \frac{1}{7}t - \frac{1}{7}, t)$ where t is any real number.

19. Find the exact solution to the following exponential equation.

$$2^{x+1} = 3^{2x-1}$$

A. $x = -\dfrac{1}{6}$ B. $x = 2$ C. $x = \dfrac{2\ln 3 - 1}{1 + \ln 2}$ D. $x = \dfrac{\ln 3 + \ln 2}{2\ln 3 - \ln 2}$ E. There is no solution.

20. Which of the following is true about the conic section whose equations is given below?

$$\frac{(x+4)^2}{4} - y^2 = 1$$

A. It is a hyperbola whose asymptotes have slopes $\pm\frac{1}{2}$

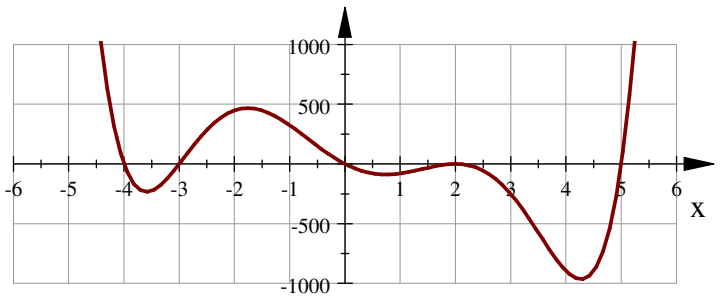
B. It is a hyperbola with vertices $(-2, 1)$ and $(-2, -1)$

C. It is a parabola with a vertex of $(-4, 0)$

D. It is a circle with radius 1

E. It is an ellipse

21. The graph of the polynomial function $P(x)$ is shown below. It has five distinct real zeros. Use it to determine which of the following statements is **false**.



A. The degree of P is at least 6 B. $P(0) = 0$ C. $(x - 5)$ is a factor of $P(x)$

D. $x = 2$ is a zero of even multiplicity E. $x = -4$ is a zero of even multiplicity

22. Determine which of the following expressions are equivalent to $\sin 4x$.

A. $4\sin x\cos x$ B. $2\sin^2 x + 2\sin x + 1$ C. $2\sin^2 x\cos^2 x$ D. $4\sin x\cos^3 x - 4\sin^3 x\cos x$ E. $4\sin x$

23. Identify the partial fraction setup for the following rational function. You do not need to find the values of the coefficients.

$$\frac{x^2(x-3)(x-4)^2}{(2x+1)(x^2+1)^3}$$

A. $\dfrac{A}{2x+1} + \dfrac{B}{(x^2+1)^3}$ B. $\dfrac{A}{2x+1} + \dfrac{Bx+C}{x^2+1} + \dfrac{Dx+E}{(x^2+1)^2} + \dfrac{Fx+G}{(x^2+1)^3}$

C. $\dfrac{A}{2x+1} + \dfrac{B}{x^2+1} + \dfrac{C}{(x^2+1)^2} + \dfrac{D}{(x^2+1)^3}$ D. $\dfrac{A}{2x+1} + \dfrac{Bx+C}{(x^2+1)^3}$

E. $\dfrac{A}{x^2} + \dfrac{B}{x-3} + \dfrac{C}{x-4} + \dfrac{D}{2x+1} + \dfrac{E}{x^2+1}$

24. What is the period of the function $f(x) = 2 + 5\sin(100x - \pi)$?

A. 5 B. $\dfrac{1}{50}$ C. 2π D. $\dfrac{\pi}{100}$ E. $\dfrac{\pi}{50}$

25. Let A, B, and C be the measures of the interior angles of an triangle. Let a, b, and c be the lengths of the sides opposite these angles, respectively. If $a = 10$, $b = 12$, and $A = 20^\circ$, which of the following statements is true?

A. There is no such triangle possible.
B. There is one possible triangle with an obtuse angle.
C. There is one possible triangle with a right angle.
D. There are two possible oblique triangles.
E. There are three possible oblique triangles.