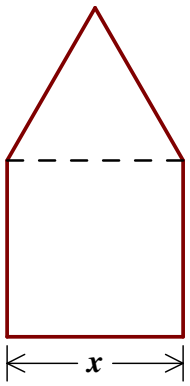
TMATYC - Precalculus Test - 2018

1. Solve for b if $a = \dfrac{\frac{2}{3}}{\frac{1}{4b} + \frac{1}{c}}$

A. $b = \dfrac{3a - c}{4}$ B. $b = \dfrac{c}{6a(c+1)}$ C. $b = \dfrac{c}{3ac - 4}$ D. $b = \dfrac{8ac}{12a + 3c}$ E. $b = \dfrac{3ac}{8c - 12a}$

2. A window is constructed so that it is in the shape of a square with an equilateral triangle on top (see figure below). Express the area of the window in terms of the side length x of the square portion.



A. $5x$ B. $\frac{3}{2}x^2$ C. $x^2 + \frac{1}{2}x$ D. $\left(\dfrac{4+\sqrt{3}}{4}\right)x^2$ E. $\left(\dfrac{2+\sqrt{3}}{2}\right)x^2$

3. Find the area of the region that lies outside the circle $x^2 + y^2 = 4$ but inside the circle $x^2 - 4x + y^2 - 12 = 0$.

A. 2π B. 4π C. 8π D. 12π E. 16π

4. The line passing through the points $(1, a)$ and $(3, b)$ has slope -2. Find the slope of the line passing through the points $(4, -a)$ and $(12, -b)$.

A. -2 B. 2 C. $-\frac{1}{2}$ D. $\frac{1}{2}$ E. Cannot be determined

5. If $f(x) = \dfrac{4}{3x+1}$ then which of the following is the simplified expression for the difference quotient

$$\frac{f(a+h) - f(a)}{h}$$

assuming $h \neq 0$?

A. $\dfrac{-12}{(3a+1)(3a+3h+1)}$ B. $\dfrac{-12}{(3a+1)^2}$ C. $\dfrac{4(3h+2)}{h(3a+1)}$ D. $\dfrac{4}{h(3h+1)}$ E. 1

6. What is the domain of the following function?

$$g(x) = \frac{\sqrt{x+300}}{x - \frac{10x}{\sqrt{400-x}}}$$

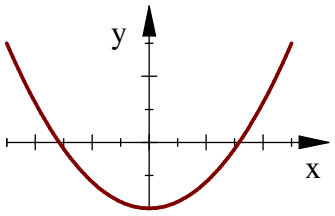
A. $[-300, 0) \cup (0, 300) \cup (300, 400)$ B. $[-300, 0) \cup (0, 400)$ C. $[-300, 400)$

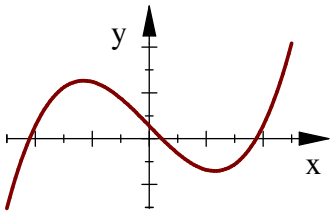
D. $\{x | x \neq 0, 300, 400\}$ E. $(-\infty, \infty)$

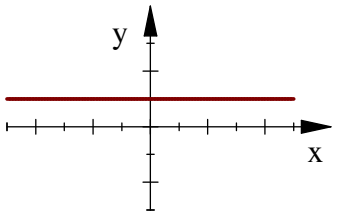
7. If $h(x) = f(f(f(x)))$ and $f(x) = 3x - 1$, then $h(1) =$

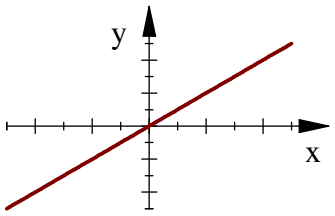
A. 26 B. 14 C. 8 D. 0 E. $\frac{1}{3}$

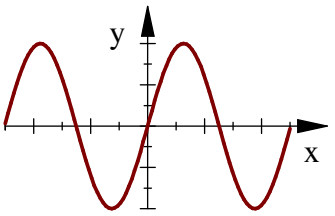
8. In each graph below, y is a function of x. Which of them is a one-to-one function?

A. 

B. 

C. 

D. 

E. 

9. If $f(x) = \dfrac{5 + \sqrt[3]{4x-1}}{7}$ then $f^{-1}(x) =$

A. $\dfrac{(7x-5)^3 + 1}{4}$ B. $\dfrac{(7x)^3 - 124}{4}$ C. $\dfrac{7}{5-(x+4)^3}$ D. $\dfrac{7}{5+\sqrt[3]{4x-1}}$ E. $7\left(5 - \left(\frac{1}{4}x + 1\right)^3\right)$

10. The graph of the quadratic function $f(x) = ax^2 + bx + c$ is a parabola that passes through the point $(2, 10)$. If its axis of symmetry is $x = -1$, then a point that is definitely on the graph is

A. $(-4, 10)$ B. $(-2, 10)$ C. $(-1, 9)$ D. $(2, 9)$ E. $(3, 11)$

11. Determine the end behavior of the polynomial function $p(x) = x^2(x-50)^2(x+50)$.

A. $y \to \infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

B. $y \to -\infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

C. $y \to \infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

D. $y \to -\infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

E. Cannot be determined

12. The polynomial $P(x) = a_{11}x^{11} + a_{10}x^{10} + \cdots + a_1x + a_0$ $(a_{11} \neq 0)$ has at most m number of x-intercepts and at least n number of x-intercepts. The sum $m + n$ is

A. 9 B. 10 C. 11 D. 12 E. 13

13. What is the value of $(\log_{624}625)(\log_{623}624)(\log_{622}623)\cdots(\log_6 7)(\log_5 6)$?

A. 2 B. 2.5 C. 4 D. 5 E. 6

14. What is the sum of the solutions to the following equation?

$$\frac{3^{2x}+14}{9} = 3^x$$

A. $\log_3 14$ B. $\log_3 12$ C. $\log_3 5$ D. 2 E. $\frac{12}{5}$

15. While driving on a cold winter day (20°F outside) your car overheats (at about 220°F). You park the car and the engine begins to cool down. The temperature T (in degrees Fahrenheit) of the engine t minutes after you park satisfies the equation

$$\ln\left(\frac{T-20}{200}\right) = -0.11t$$

What is the temperature of the engine (to the nearest degree Fahrenheit) 20 minutes after you park?

A. 89 B. 60 C. 49 D. 42 E. 35

16. The point P lies on the unit circle (i.e. the circle of radius one with center at the origin). If P lies in quadrant II and its y-coordinate is $\frac{2\sqrt{3}}{5}$ then its x-coordinate is

A. $\frac{12}{25}$ B. $\frac{\sqrt{13}}{5}$ C. $\frac{\sqrt{19}}{5}$ D. $-\frac{\sqrt{13}}{5}$ E. $-\frac{\sqrt{19}}{5}$

17. If $\cot t = \frac{2}{3}$ and t lies in quadrant III, then $\sin t =$

A. $\frac{3}{13}$ B. $\frac{3\sqrt{13}}{13}$ C. $-\frac{3\sqrt{13}}{13}$ D. $-\frac{2\sqrt{13}}{13}$ E. $-\frac{3}{5}$

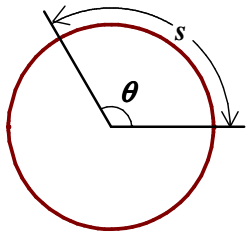
18. What is the product of the amplitude and period for the graph of $y = 500\cos(200\pi t)$

A. 5 B. 40 C. 5,000 D. $\frac{5}{2\pi}$ E. $10,000\pi$

19. In an analog clock, through how many radians **combined** do the minute hand and the hour hand move between 1:00 P.M. and 1:45 P.M. (on the same day)?

A. $\frac{3\pi}{2}$ B. $\frac{13\pi}{8}$ C. $\frac{12\pi}{9}$ D. $\frac{39\pi}{40}$ E. $\frac{13\pi}{16}$

20. Find the arclength s of the circle of radius 5 shown below if $\theta = 120°$.



A. 24 B. 600 C. $\frac{19\pi}{6}$ D. $\frac{4\pi}{3}$ E. $\frac{10\pi}{3}$

21. A giant redwood tree casts a shadow 550 feet long. Find the height of the tree if the angle of elevation to the sun is 27°. Round your answer to the nearest foot.

A. 249 B. 280 C. 490 D. 1079 E. 1800

22. Write $\tan(\cos^{-1}x)$ as an algebraic expression for $-1 \le x \le 1$.

A. $\frac{1}{x}$ B. $\frac{\sqrt{1-x^2}}{x}$ C. $\frac{x}{\sqrt{1-x^2}}$ D. $\frac{1}{\sqrt{1-x^2}}$ E. $\sqrt{1-x^2}$

23. Let $i = \sqrt{-1}$. If $\frac{a+3i}{1+2i} = 2 - i$ then $a =$

A. $-4i$ B. $-5i$ C. 2 D. 3 E. 4

24. Which of the following can be the polar coordinates of the point that has rectangular coordinates $(5,-5)$?

A. $\left(5\sqrt{2}, \frac{\pi}{4}\right)$ B. $\left(5\sqrt{2}, \frac{3\pi}{4}\right)$ C. $\left(-5\sqrt{2}, \frac{\pi}{4}\right)$ D. $\left(-5\sqrt{2}, \frac{3\pi}{4}\right)$ E. $\left(5\sqrt{2}, \frac{5\pi}{4}\right)$

25. Triangle ABC has sides of length 6, 7, and 8. Find the exact value of $\cos A + \cos B + \cos C$

A. $\frac{51}{35}$ B. $\frac{47}{32}$ C. $\frac{31}{21}$ D. $\frac{49}{33}$ E. $\frac{119}{80}$