Section 3-2

Measures of Center

Measures of Center

• Measures of center (or measures of central tendency) are values at the center or middle of a data set.

Arithmetic Mean

• The arithmetic mean (or just mean) is the measure of center obtained by adding the values and dividing the total by the number of values.
Notation

Σ denotes the addition of a set of values

\( x \) is the variable usually used to represent the individual data values

\( n \) represents the number of values in a sample

\( N \) represents the number of values in a population

Notation

\( \bar{x} \) is pronounced ‘x-bar’ and denotes the mean of a set of sample values

\[ \bar{x} = \frac{\sum x}{n} \]

\( \mu \) is pronounced ‘mu’ and denotes the mean of all values in a population

\[ \mu = \frac{\sum x}{N} \]

Median

• The median is the middle value when the original data values are ranked (i.e. arranged in order of increasing (or decreasing) magnitude).
Finding the Median

• If the number of values is odd, the median is the number located in the exact middle of the ranked list.

• If the number of values is even, the median is found by computing the mean of the two middle numbers in the ranked list.

Mode

• The mode is the value that occurs most frequently.

• The mode is the only measure of central tendency that can be used with nominal data.

Mode

• The mode may not be unique. There may be:
  - two modes (bimodal);
  - three or more modes (multimodal);
  - or no mode
Midrange

• The **midrange** is the value midway between the highest and lowest values in the original data set. It is found by taking the **mean** of the **highest** and **lowest** value.

### Best Measure of Center?

<table>
<thead>
<tr>
<th>Measure of Center</th>
<th>Definition</th>
<th>How Commonly Used</th>
<th>How Often Exists</th>
<th>Can Any Value Be Ignored?</th>
<th>Pros and Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>$x = \frac{\sum x}{n}$</td>
<td>most familiar “average”</td>
<td>always exists</td>
<td>yes</td>
<td>used throughout the book, works well with many statistical methods</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>middle value</td>
<td>commonly used</td>
<td>always exists</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td><strong>Mode</strong></td>
<td>most frequent data value</td>
<td>sometimes used</td>
<td>might not exist, may be one mode</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td><strong>Midrange</strong></td>
<td>high − low</td>
<td>rarely used</td>
<td>always exists</td>
<td>yes</td>
<td>very sensitive to extreme values</td>
</tr>
</tbody>
</table>


### Round-off Rule for Measures of Center

• Carry one more decimal place than is present in the original set of values. Exception: the mode can be left without any rounding.
Weighted Mean

In some cases, values vary in their degree of importance, so they are weighted \((w)\) accordingly

\[
\bar{x} = \frac{\sum (w \cdot x)}{\sum w}
\]

Distribution Shapes

- **Uniform**: All values appear with about the same frequency.

- **Symmetric**: Left half of histogram is essentially the mirror image of the right half.

Distribution Shapes

- **Normal**: A symmetric distribution in which the distribution actually looks like a bell.

- **Bi-modal**: The distribution has two distinct modes that appear as two “peaks”.
Distribution Shapes

- Skewed to the left (or negatively skewed): Mean and median are to the left of the mode. Data extends to the left.

- Skewed to the right (or positively skewed): Mean and median are to the right of the mode. Data extends to the right.

Figure 2-11

Skewness