Section 3-4

Measures of Relative Standing & Boxplots

Measures of Relative Standing

• Measures of relative standing (or measures of position) indicate the position of a single data value relative to the rest of the data set.

<table>
<thead>
<tr>
<th>On which test did you do better on?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Test 1</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Mean:  70.0</td>
</tr>
<tr>
<td>Standard Deviation:  5.0</td>
</tr>
<tr>
<td>Your Score:  80</td>
</tr>
</tbody>
</table>
On which test did you do better on *relative to the rest of the class*?

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>70.0</td>
<td>71.7</td>
</tr>
<tr>
<td>SD</td>
<td>5.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Your Score</td>
<td>80</td>
<td>78</td>
</tr>
</tbody>
</table>

*z Score*

- The *z score* (or *standard score*) is the number of standard deviations that a given value $x$ is above (+) or below (−) the mean.

- The *z score* is positive if the value is above the mean and negative if the value is below the mean.

$$z = \frac{x - \bar{x}}{s} \quad z = \frac{x - \mu}{\sigma}$$

Round to 2 decimal places.
On which test did you do better on relative to the rest of the class?

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Interpreting $z$ Scores

- Using the range rule of thumb we can say the following:
- “Usual” values have $z$ scores between -2 and 2.
- “Unusual” values have $z$ scores greater than 2 or less than -2.

Quartiles, Deciles, & Percentiles

- Three quartiles (designated $Q_1$, $Q_2$, and $Q_3$) divide the data into four equal parts of 25% each.
- Nine deciles (designated $D_1$, $D_2$, …, $D_9$) divide the data into ten equal parts of 10% each.
- 99 percentiles (designated $P_1$, $P_2$, …, $P_{99}$) divide the data into 100 equal parts of 1% each.
Percentiles

- \( P_1 \) (1st Percentile) separates the bottom 1% of sorted values from the top 99%.
- \( P_2 \) (2nd Percentile) separates the bottom 2% of sorted values from the top 98%.
- ...
- \( P_{99} \) (99th Percentile) separates the bottom 99% of sorted values from the top 1%.

Quartiles

- \( Q_1 \) (First Quartile) separates the bottom 25% of sorted values from the top 75%.
- \( Q_2 \) (Second Quartile) separates the bottom 50% of sorted values from the top 50%.
- \( Q_3 \) (Third Quartile) separates the bottom 75% of sorted values from the top 25%.

Relationship between Quartiles and Percentiles

Note that

\[ Q_1 = P_{25} \]
\[ Q_2 = P_{50} = \text{Median} \]
\[ Q_3 = P_{75} \]
The Interquartile Range (IQR)

- The interquartile range (or IQR) is defined to be
  \[ Q_3 - Q_1 \]
  the difference between the first and third quartile.

5-Number Summary

- For a set of data, the **5-number summary** consists of the minimum value; the first quartile, \( Q_1 \); the median (or second quartile, \( Q_2 \)); the third quartile, \( Q_3 \); and the maximum value.

Boxplot

- A **boxplot** (or box-and-whisker-diagram) is a graph of a data set that consists of a line extending from the minimum value to the maximum value, and a box with lines drawn at the first quartile, \( Q_1 \); the median; and the third quartile, \( Q_3 \).
Boxplots

Figure 2-17