Section 5-3
Binomial Probability Distributions

Definitions
A binomial probability distribution results from a binomial experiment:
1. The procedure has a fixed number of trials.
2. The trials must be independent. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into two categories we call success and failure.
4. The probabilities of success and failure must remain constant for each trial.

Binomial Experiment?
• Asking 30 1st graders for their favorite color
• Asking 30 1st graders if they like red
• Rolling a fair six-sided die 100 times to see what you get each time
• Rolling a fair six-sided die 100 times to see when you get a 6.
• Rolling a loaded six-sided die 100 times to see when you get a 6.
Notation (cont)

\( n \)\quad \text{denotes the number of fixed trials.}

\( x \)\quad \text{denotes a specific number of successes in } n \text{ trials, so } x \text{ can be any whole number between 0 and } n, \text{ inclusive.}

\( p \)\quad \text{denotes the probability of success in one of the } n \text{ trials.}

\( q \)\quad \text{denotes the probability of failure in one of the } n \text{ trials.}

\( P(x) \)\quad \text{denotes the probability of getting exactly } x \text{ successes among the } n \text{ trials.}

Important Hints

Always write out (yes write) what your trials are, what your two outcomes are for each trial, and which outcome you are calling success.

• Be sure that \( x \) and \( p \) both refer to the same category being called a success.

• When sampling without replacement, the events can be treated as if they were independent if the sample size is no more than 5\% of the population size. (That is \( n \) is less than or equal to 0.05\( N \).)

Methods for Finding Probabilities

We will now present three methods for finding the probabilities corresponding to the random variable \( x \) in a binomial distribution.
Method 1: Using the Binomial Probability Formula

\[ P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x} \]

for \( x = 0, 1, 2, \ldots, n \)

where
\( n \) = number of trials
\( x \) = number of successes among \( n \) trials
\( p \) = probability of success in any one trial
\( q \) = probability of failure in any one trial \((q = 1 - p)\)

Rationale for the Binomial Probability Formula

\[ P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x} \]

Number of outcomes with exactly \( x \) successes among \( n \) trials

Binomial Probability Formula

\[ P(x) = \frac{n!}{(n-x)!x!} \cdot p^x \cdot q^{n-x} \]

Number of outcomes with exactly \( x \) successes among \( n \) trials
Probability of \( x \) successes for any one particular order
Binomial Probability Formula

I will use the following notation in the formula:

\[ P(x) = \binom{n}{x} \cdot p^x \cdot q^{n-x} \]

If 79.5% of all statistics students are registered voters, what is the probability that 10 of 12 randomly selected statistics students are registered voters?

It is known that 20% of all PSTCC students take a statistics course. If four PSTCC students are randomly selected, what is the probability that exactly three of will take a statistics course?
Method 2: Using Table A-1 in Appendix A

Part of Table A-1 is shown below. With $n = 4$ and $p = 0.2$ in the binomial distribution, the probabilities of 0, 1, 2, 3, and 4 successes are 0.410, 0.410, 0.154, 0.026, and 0.002 respectively.

<table>
<thead>
<tr>
<th>From Table A-1:</th>
<th>$n$</th>
<th>$x$</th>
<th>$p(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>4</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>4</td>
<td>0.410</td>
<td>0.410</td>
</tr>
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</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.154</td>
<td>0.154</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

It is known that 30% of all first graders like the color red. If we randomly select 12 first graders, what is the probability that at least 4 like the color red?

What is the probability less than 4 of the 12 like the color red?

Example

(Adapted from #9 on page 138)

A psychology professor gives a surprise quiz consisting of 10 true/false questions, and she states that a score of 70% is passing. If an unprepared student randomly guesses at each question, what is the probability they pass the quiz?