Section 7-4
Estimating a Population Mean: σ Not Known

σ Not Known Assumptions
1. The sample is a simple random sample.
2. σ not known.
3. Either the sample is from a normally distributed population, or \( n > 30 \).

σ Not Known Assumptions
Under these assumptions, we will be using a new distribution for finding critical values. It is called the Student \( t \)-distribution or just \( t \)-distribution.
Important Properties of the Student $t$ Distribution

1. The Student $t$ distribution is different for different sample sizes (see Figure 7-5 for the cases $n = 3$ and $n = 12$).

2. The Student $t$ distribution has the same general symmetric bell shape as the normal distribution but it reflects the greater variability (with wider distributions) that is expected with small samples.

3. As the sample size $n$ gets larger, the Student $t$ distribution gets closer to the normal distribution.

Important Properties of the Student $t$ Distribution

1. The Student $t$ distribution has a mean of $t = 0$ (just as the standard normal distribution has a mean of $z = 0$).

2. The standard deviation of the Student $t$ distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a $\sigma = 1$).

3. As the sample size $n$ gets larger, the Student $t$ distribution gets closer to the normal distribution.

Student $t$ Distributions for $n = 3$ and $n = 12$
Definition

Degrees of Freedom ($df$) corresponds to the number of sample values that can vary after certain restrictions have been imposed on all data values.

$$df = n - 1$$

in this section.

Margin of Error $E$ for Estimate of $\mu$

Based on an Unknown $\sigma$

Formula 7-6

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ has $n - 1$ degrees of freedom.

Confidence Interval for the Estimate of $E$

Based on an Unknown $\sigma$

$$\bar{x} - E < \mu < \bar{x} + E$$

where $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$

t_{\alpha/2} found in Table A-3
Choosing between the \( z \) and \( t \) Distributions

Sample Size for Estimating Mean \( \mu \)

\[
n = \left( \frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2
\]

Round-Off Rule for Sample Size \( n \)

When finding the sample size \( n \), if the use of Formula 7-5 does not result in a whole number, always increase the value of \( n \) to the next larger whole number.
Finding the Sample Size $n$ when $\sigma$ is unknown

1. Use the range rule of thumb (see Section 3-3) to estimate the standard deviation as follows: $\sigma \approx \frac{\text{range}}{4}$.

2. Conduct a pilot study by starting the sampling process. Based on the first collection of at least 31 randomly selected sample values, calculate the sample standard deviation $s$ and use it in place of $\sigma$.

3. Estimate the value of $\sigma$ by using the results of some other study that was done earlier.