1. There are a total of $16 + 12 + 8 + 4 = 40$ marbles in the bag.
   a. $\frac{16}{40} = \frac{2}{5} = 0.4$
   b. There are 36 marbles that are NOT green. Thus the probability is $\frac{36}{40} = \frac{9}{10} = 0.9$
   c. Since we are drawing WITH replacement, each drawing is INDEPENDENT. Thus,
      $P(\text{both white}) = P(\text{white AND white}) = P(\text{white}) \times P(\text{white}) = \left(\frac{12}{40}\right) \times \left(\frac{12}{40}\right) = \frac{144}{1600} = 0.09$
   d. WITHOUT replacement means the draws are NOT independent. We can still multiply
      the probabilities together, but we make sure the probability the second is white
      assumes that the first one drawn was white. $P(\text{white and white}) = \left(\frac{12}{40}\right) \times \left(\frac{11}{39}\right) = \frac{132}{1560} = \frac{11}{130} \approx 0.0846$
   e. $\left(\frac{16}{40}\right) \times \left(\frac{12}{39}\right) \times \left(\frac{8}{38}\right) \times \left(\frac{15}{37}\right) \approx 0.0105$

2. Here BOTH selected people have to be born in January.
   $P(\text{both born in Jan}) = P(1\text{st born in Jan AND 2nd born in Jan})$
   $\quad = \left(\frac{31}{365}\right) \times \left(\frac{31}{365}\right) = \frac{961}{133,225} \approx 0.00721$

3. We can find the probability of the event by finding the probability of its complement.
   Recall that there are 31 days in October and 30 days in November.
   $P(\text{Not born in Oct or Nov}) = 1 - P(\text{Born in Oct or Nov})$
   $\quad = 1 - \frac{61}{365} = \frac{304}{365} \approx 0.833$

4. I include the row and column totals for the table below

<table>
<thead>
<tr>
<th></th>
<th>Democrat</th>
<th>Republican</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>13</td>
<td>136</td>
<td>1</td>
</tr>
<tr>
<td>No</td>
<td>149</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>Abstain</td>
<td>3</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>165</td>
<td>160</td>
<td>10</td>
</tr>
</tbody>
</table>

   Note that we can now tell, for instance, that there were 165 Democrats who voted.

   a. There were 160 republicans so $\frac{160}{335} \approx 0.478$
   b. There were 170 no votes so $\frac{170}{335} \approx 0.507$. 
   c. We count all Democrats (165) and all yes votes (150), but we need to deduct the 13
      Democrats who voted yes since this was counted twice. Thus there are 165 + 150 - 13
      = 302 Democrats or yes votes. Thus the probability is $\frac{302}{335} \approx 0.901$.
   d. There 13 Democrats who voted yes. Thus the probability is $\frac{13}{335} \approx 0.0388$

5. Part (a) can be done with the TI-83 by placing the $x$ values in L1 and corresponding $\mathbb{P}(x)$
   values in L2 and using $\mathbf{1-Var Stats}$.

   a. $\mu \approx 3.3$ and $\sigma \approx 1.8$
   b. $\mu - 2\sigma = 3.3 - 2(1.8) = -0.3$ and $\mu + 2\sigma = 3.3 + 2(1.8) = 6.9$. Thus anything
      between -0.3 (or 0) and 6.9 would be usual. Since 6 falls in this interval, giving out 6
      tickets would NOT be unusual.
6. This is a Binomial Experiment. The trials are sampling an E.G. lightbulb to see if it works. The outcomes are the lightbulb is defective or it is not defective. Success is that the lightbulb is defective. We have \( n = 20, p = 0.03 \) (3%), \( x = 2 \). So the probability is

\[
P(x = 2) = 20 \binom{2}{2} \cdot (0.03)^2 \cdot (0.97)^{18} \approx 0.0988
\]

7. The trials are randomly guessing at ONE question. The outcomes are you guess correctly or you guess incorrectly. Let success be that you guess correctly. Then \( n = 8 \) and \( p = 1/4 = 0.25 \) (since only one of the four possible answers are correct). Now the probability you pass is the probability you get 60% OR MORE. 60% of 8 is 0.6(8) = 4.8. Thus we need to answer 5 or more correctly to pass (\( x \geq 5 \))

\[
P(x \geq 5) = P(5) + P(6) + P(7) + P(8) = ^8C_5 \cdot (0.25)^5 \cdot (0.75)^3 + ^8C_6 \cdot (0.25)^6 \cdot (0.75)^2 + ^8C_7 \cdot (0.25)^7 \cdot (0.75)^1 + ^8C_8 \cdot (0.25)^8 \cdot (0.75)^0
\]

\[
eq 0.0230712891 + 0.0038452148 + 0.0003662109375 + 0.00001525878906 \approx 0.0273
\]

This could also be found on the TI-83 with \( 1 - \text{binomcdf}(8, .25, 4) \).

It is NOT likely that you will pass if you randomly guess.

8. A bag contains 40 marbles all the same size. Sixteen are red, fourteen are black, and ten are white.

a. \( 16/40 = 0.4 \)

b. Note that by doing this with replacement we have a binomial experiment. The trials are selecting a marble. The outcomes are red or not red. Let success be that the marble is red. Thus \( n = 12 \) and \( p = 0.4 \) (see part (a)).

i. \( P(x = 4) = 0.213 \)

ii. \( P(x < 4) = P(3) + P(2) + P(1) + P(0) = .142 + .064 + .017 + .002 = 0.225 \) (Using Table A-1)

iii. \( P(x \geq 10) = P(10) + P(11) + P(12) = .002 \) (Using Table A-1)

c. YES, since the probability is so low (only 0.2%)

9. Binomial experiment. Trials are randomly selecting a college freshman and the outcomes are they do know or they do not know the name of the capital of Maine. Thus \( n = 10 \) in each situation below.

a. Here success is that they do NOT know the name of the capital of Maine. Since we are told that 20% of freshmen know the name of the capital, we must have 80% who do NOT know the name. Thus, for this situation, \( p = 80\% = 0.8 \). We have eight successes so that \( x = 8 \). Table A-1 can be used or by the formula/calculator:

\[
P(\text{eight do NOT know the capital of Maine}) = P(x = 8) = 10C_8 \cdot (0.8)^8 \cdot (0.2)^2 \approx 0.302
\]

b. Here success is that they KNOW the name of the capital. Thus \( p = 20\% = 0.2 \). We have \( x \geq 3 \) or \( x = 3, 4, 5, 6, 7, 8, 9, \) or 10. Using Table A-1 we have

\[
\]

\[
eq 0.201 + 0.088 + 0.026 + 0.006 + 0.001 + 0 + 0 + 0 = 0.322
\]

or
\[ P(\text{at least three know the capital of Maine}) = P(x \geq 3) = 1 - P(x < 3) \]
\[ = 1 - (P(0) + P(1) + P(2)) \]
\[ = 1 - (0.107 + 0.268 + 0.302) = 0.323 \]

The differences are due to rounding in Table A-1.

10. A multiple choice exam has 100 questions each with four possible answers.

a. We have a binomial experiment. The trials are randomly guessing at a question. The outcomes are guessing correctly or guessing incorrectly. Success is guessing correctly. Thus \( n = 100 \) and \( p = 1/4 = 0.25 \) (so \( q = 0.75 \)). Thus

\[ \mu = np = (100)(.25) = 25 \]
\[ \sigma = \sqrt{npq} = \sqrt{(100)(.25)(.75)} \approx 4.3 \]

b. 

Maximum : \( \mu + 2\sigma = 25 + 2(4.3) = 33.6 \)
Minimum : \( \mu - 2\sigma = 25 - 2(4.3) = 16.4 \)

Thus you should expect to guess between about 16 and 34 correctly.