2.3 Polynomial Functions

Terminology

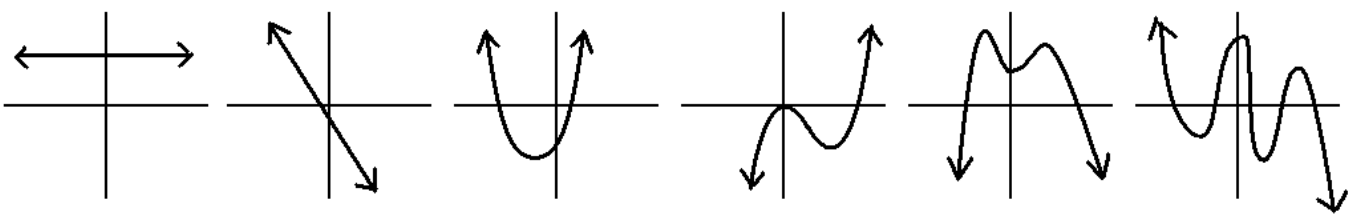
A **polynomial** can be expressed in its term-by-term form (unfactored).
The standard is to write terms in decreasing order of powers of *x*.

$f(x) = 4x^2 - 4x - 15$ or $g(x) = -6x^5 + 23x^4 + 5x^3 - 4x^2$

They can also be represented in a factored or product form (if it can be factored)

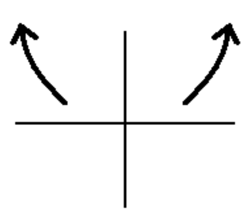
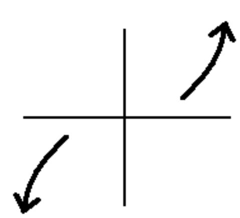
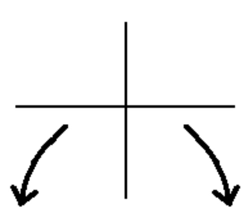
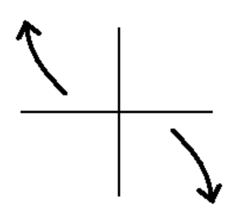
$f(x) = (2x+3)(2x-5)$ or $g(x) = -x^2(3x-1)(x-4)(2x+1)$

- The variable *x* can only be raised to positive whole number exponents.
- Polynomials are characterized by their **degree: the highest exponent on *x***
- **The domain of a polynomial is ALWAYS all real numbers:** $(-\infty, \infty)$.
- Polynomial graphs are smooth and connected without any breaks or gaps.
- **A polynomial's graph can have AT MOST 1 fewer turning points than its degree.**



degree = 0	degree = 1	degree = 2	degree = 3	degree = 4	degree = 5
'constant'	'linear'	'quadratic'	'cubic'	'quartic'	'quintic'

The **<u>END BEHAVIOR</u>** of a polynomial function depends on its degree and the sign of its leading coefficient. The is called the **<u>LEADING COEFFICIENT TEST</u>**

	Even Degree	**Odd Degree**
Positive Leading Coefficient		
Negative Leading Coefficient		

ex) Determine the end behavior of the following polynomials. State your answer as an end behavior sketch.

a) $f(x) = 18x - 9x^2 - 2x^3$

b) $g(x) = x^2(3x+1)(x-6)$

Zeros of Polynomials

A **zero** for a polynomial is any *x* value (real OR complex) which gives a '0' y-output. All zeros are found by setting $y = 0$ and solving for x. (Usually involves factoring.) A zero is characterized by its **multiplicity**: the power of the factor giving that zero.

ex) Determine the zeros for the polynomials below.

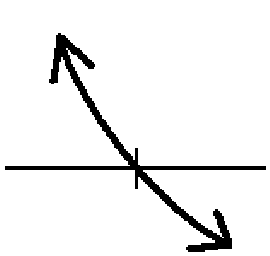
a) $f(x) = 18x - 9x^2 - 2x^3$

b) $g(x) = x^2(3x+1)(x-6)$

The multiplicity of a zero will affect the way it crosses the *x*-axis. Each zero's multiplicity comes from the factored form.

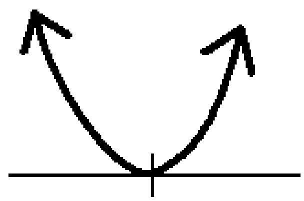
Multiplicity of a zero:

1



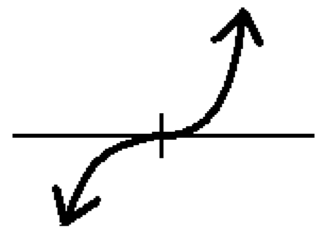
Graph will go *through* x-axis relatively straight.

2



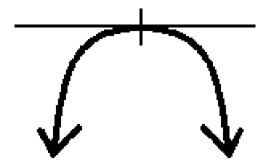
Graph will *touch* the x-axis and double back.

3



Graph will go *through* the x-axis relatively flat.

4



Graph will *touch* the x-axis relatively flat

ex) Determine x-intercept behavior for these polynomials.

a) $f(x) = 18x - 9x^2 - 2x^3$

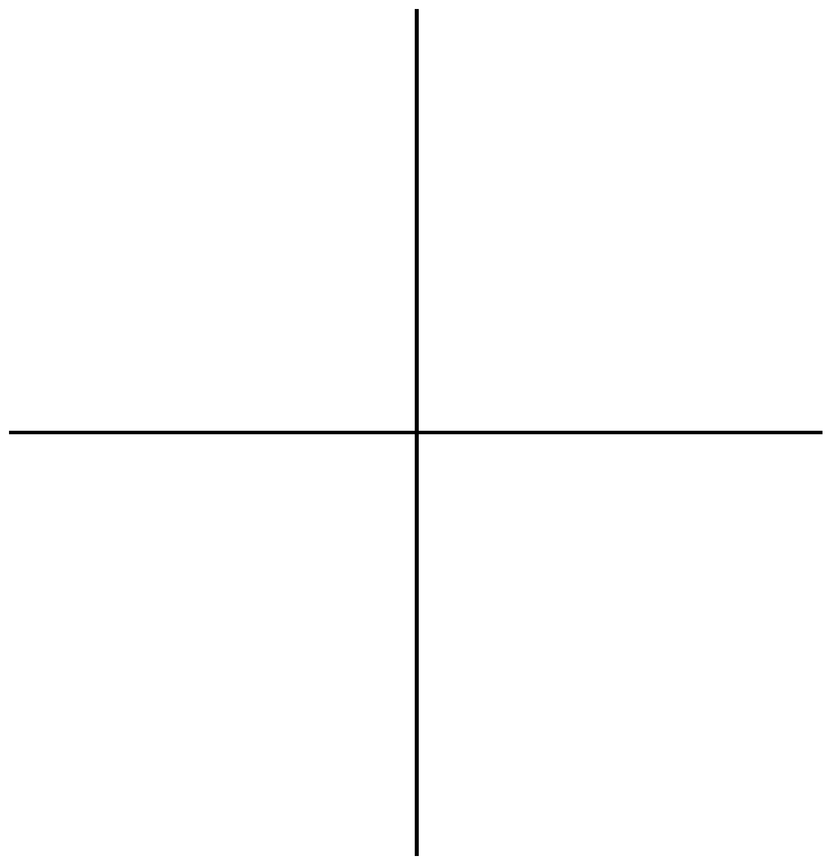
b) $g(x) = x^2(3x+1)(x-6)$

Sketching a Polynomial's Graph

- Determine the graph's end behavior (L.C.T.)
- Locate the zeros and their multiplicities
- Make a rough sketch based on this information; then refer to the calculator to determine the **relative maximum** and **relative minimum** to help scale your graph.

ex) Sketch the graph of the polynomial:

$$f(x) = -x^3 - 2x^2 + 4x + 8$$



ex) Sketch the graph of the polynomial:

$$g(x) = x(x-3)(x+4)^2$$