Ex. What do the y-values of the graph of $f(x) = \frac{\sin x}{x}$ approach as the x-values approach 0?

Look at a table of values for the function as $x \to 0$ (use ASK mode)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.99983</td>
</tr>
<tr>
<td>0.01</td>
<td>0.99998</td>
</tr>
<tr>
<td>-1</td>
<td>0.99983</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.99998</td>
</tr>
</tbody>
</table>

Look at the graph as $x \to 0$ (ZOOMDEC)

Based on the numerical data and the graphical data the graph seems to approach $y = 1$ as $x \to 0$.

But, the function is not defined at $x = 0$.

Look at the graph with the axes turned off. (2\textsuperscript{nd} FORMAT, AxesOff)

You can barely tell, but there is a hole in the graph at $x = 0$. The function is not defined there.

**DEFINITION OF LIMIT**: We write

$$\lim_{x \to a} f(x) = L$$

and say “the limit of $f(x)$ as $x$ approaches $a$, equals $L$”

if we can make the values of $f(x)$ arbitrarily close to $L$ (as close to $L$ as we like) by taking $x$ to be sufficiently close to equal $a$.

When evaluating limits, you’re actually finding the “intended value” of the function as it approaches a certain x-value.

In the above example the function $f(x) = \frac{\sin x}{x}$ is not defined at $x = 0$, but we can say that the limit as $x$ approaches 0 does equal 1 because that is the “intended” y-value the graph approaches.

So $\lim_{x \to 0} \frac{\sin x}{x}$ does exist and $\lim_{x \to 0} \frac{\sin x}{x} = 1$, regardless of whether or not the function is defined there.
Ex. Evaluate these limits either graphically or numerically (using your table):

\[ a) \lim_{x \to 2} 2x^2 - 3x + 5 \quad b) \lim_{x \to 1} \frac{x^2 - 3x - 4}{x + 1} \]

**SOLUTION:***

a) There’s no trouble just plugging in the value \( x = 2 \) into the function. This would give us the value of the limit as \( x \to 2 \). You should get a limit of \( L = 7 \).

b) If you consider a table of values, you notice you can’t simply plug in \( x = -1 \) because the function isn’t defined there, but as the table suggests, the \( y \)-values approach a limit of \( -5 \), so

\[ \lim_{x \to -1} \frac{x^2 - 3x - 4}{x + 1} = -5 \]

**Left hand and Right hand limits:** When using your table to evaluate these limits you’ve considered values to the left and to the right of the central \( x \) value you’re approaching. These are called the left hand and right hand limits of the function \( f(x) \).

\[ \lim_{x \to a^-} f(x) = L \quad \text{taking values of } x \text{ approaching } a \text{ from the left, } x < a \]

\[ \lim_{x \to a^+} f(x) = L \quad \text{taking values of } x \text{ approaching } a \text{ from the right, } x > a \]

**IMPORTANT!** The limit \( \lim_{x \to a} f(x) = L \) exists ONLY if \( \lim_{x \to a^-} f(x) = L \) AND \( \lim_{x \to a^+} f(x) = L \)

Ex. Given the following graph evaluate the following quantities:

Ex. Sketch a graph of an example of a function \( f(x) \) that satisfies the following conditions:

\[ \lim_{x \to -1} f(x) = 4 \quad \lim_{x \to 1} f(x) = -2 \quad f(-1) \text{ is undefined} \]

\[ \lim_{x \to 3} f(x) = 0 \quad f(3) = 6 \]