Rogawski 3.10 - Related Rates – Using techniques of implicit differentiation to solve application problems

Suggested solution method:
1. Read problem carefully.
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary use the geometry of the situation to eliminate one of the variables by substitution.

Now try my FUN Related Rates Introductory Problems...

1. a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.
   b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30m?

2. A particle moves along the curve \( y = \sqrt{1 + x^3} \). As it reaches the point (2,3), the y coordinate is increasing at a rate of 4 cm/s. How fast is the x-coordinate of the point changing at that instant?

3. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4PM?

4. A streetlight is mounted at the top of a 15 ft pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

5. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

6. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

7. Water is leaking out of an inverted conical tank at a rate of 10,000 cm³/min at the same time that water is being pumped into the tank at a constant rate. The tank has a height of 6 m and a diameter of 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.

8. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the ground (the kite is anchored to the ground) decreasing when 200 ft of string have been let out?

9. Two sides of a triangle are 12 m and 15 m in length. The angle between them is increasing at a rate of 2°/min. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°?

10. Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?