The only techniques we know so far is term-by-term differentiation.

Ex) The derivative of \( y = x^3 + e^x \) is

So, as of now, we have no known means for finding the derivative for a function like \( y = x^3 e^x \) which is made up of a **product of two functions using the variable** \( x \).

Functions formed as products or quotients of other functions **CANNOT** be differentiated factor-by-factor.

**THE PRODUCT RULE:**

For functions in the form \( y = f(x) \cdot g(x) \), the derivative is \( y' = f'(x) \cdot g(x) + f(x) \cdot g'(x) \)

Ex) Find the derivative for the function \( y = x^3 e^x \).
Ex) Find \( \frac{dy}{dx} \) for \( y = (2x^2 + 3x - 5)(x^2 + 6) \).

Ex) Find the derivative for \( y = \frac{e^x}{x} \).
THE QUOTIENT RULE:

For functions in the form \( y = \frac{f(x)}{g(x)} \), the derivative is \( y' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2} \).

Ex) Find the derivative for \( y = \frac{e^x}{x} \) using the quotient rule.

Ex) Find the derivative for the function \( f(x) = \frac{x^2 + 1}{2x + 3} \).
Ex) Find the equation of the tangent line to the function \( f(x) = \frac{2x^2 - 4x + 1}{x^2 + 5x + 3} \) at \( x = -1 \).