Section 5.2 - Diagonalization

If $A$ and $B$ are both $n \times n$ matrices such that there exists an $n \times n$ invertible matrix $P$ such that

$$B = P^{-1}AP$$

we say $B$ is similar to $A$.

If $B$ is similar to $A$, then $A$ is similar to $B$.

**Proof:** Since $B$ is similar to $A$

$$B = P^{-1}AP$$

$$PB = P(P^{-1}AP) = (PP^{-1})AP = I(AP) = AP$$

$$PBP^{-1} = (AP)P^{-1} = A(PP^{-1}) = AI = A$$

Now let $Q = P^{-1}$, then we have $Q^{-1} = (P^{-1})^{-1} = P$ and

$$A = Q^{-1}BQ$$

so that $A$ is similar to $B$.

Similar matrices share many properties. For instance, they have the same determinant:
A square matrix $A$ is said to be diagonalizable if it is similar to a diagonal matrix $D$ (i.e. there is an invertible matrix $P$ such that $P^{-1}AP = D$)

**Theorem:** If $A$ is $n \times n$, then the following are equivalent:

(a) $A$ is diagonalizable

(b) $A$ has $n$ linearly independent eigenvectors.

**Theorem:** If $\lambda_1, \lambda_2, \ldots, \lambda_k$ are distinct eigenvalues of matrix $A$, and if $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ are corresponding eigenvectors, then $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is a linearly independent set.

**Theorem:** An $n \times n$ matrix with $n$ distinct eigenvalues is diagonalizable.

Procedure for diagonalizing an $n \times n$ matrix: See page 305 of textbook.
Diagonalize the matrix $A = \begin{bmatrix} 1 & -8 \\ -4 & -3 \end{bmatrix}$